Exam for Random Processes, 5 points/7.5 ECTS

March 15, 2005, 1.30 pm – 5.30 pm

Max number of points: 30.
Halmstad University grading bounds: 12p ⇒ grade 3, 18p⇒ grade 4, 24p ⇒ grade 5.
ECTS bounds: 12p ⇒ grade E, 15p ⇒ grade D, 18p ⇒ grade C, 21p ⇒ grade B, 24p ⇒ grade A.
Allowed aids: Sheet of formulae attached to the exam, calculator and Mathematics Handbook: Beta.
Examiner: Eric Järpe (035-16 76 53).

For each problem a *complete* solution should be given. All solutions should be thoroughly presented. Each solution should start at the top of a new sheet of paper. Only one solution a sheet. The proper solutions will be available on the internet at
`http://www.hh.se/staff/erja` → `Teaching` → `Random processes` → `050315: Solution`

1. Show that if $\{X_t : t \in \mathbb{R}\}$ is weakly stationary, $Z_t = X_t$ for $t = 0, \pm\frac{1}{2}, \pm 1, \pm\frac{3}{2}, \ldots$, then
$$R_Z(f) = \sum_{k\in\mathbb{Z}} R_X(f + 2k) \quad \text{where } -1 < f \le 1 \qquad (4p)$$

2. Show the Yule-Walker equations, i.e. if $\{X_t\}$ is an $AR(p)$ process, then
$$\sum_{k=0}^{p} a_k r_X(\tau - k) = \begin{cases} \sigma_\epsilon^2 & \text{for } \tau = 0 \\ 0 & \text{for } \tau = 1, 2, \ldots \end{cases} \qquad (3p)$$

3. A car drives on a trip for several hours. During hour k it drives with speed X_k km/h independently of speed during previous hours, where $k = 1, 2, 3, \ldots$, and $X_k \in Poi(50)$. Let Y_t be the distance driven by time t where t is measured in hours from the start. Calculate
   (a) $V(X_4)$. (2p)
   (b) $C(Y_3, Y_4)$. (4p)
   (c) approximately $P(200 \le Y_4 \le 220)$. (2p)

4. Assume $\{X_t : t \in \mathbb{R}\}$ is weakly stationary and has cvf $r_X(\tau) = 1/(1 + \tau^2)$. Determine the cross-cvf with the derivative process $r_{X,X'}(s, t)$. (3p)

5. An $MA(2)$ process $\{X_t : t \in \mathbb{Z}\}$ is defined by $X_t = \epsilon_t + \epsilon_{t-2}$ where $V(\epsilon_t) = 1$. Calculate its spectral density function $R_X(f)$. (4p)

6. Suppose $\{X_t : t \in \mathbb{R}\}$ with spectral density $e^{-|f|}$ is filtered with impulse response $\delta_{-1}(t) + \delta_1(t)$. What is the spectral density $R_Y(f)$ of the output signal $\{Y_t\}$?(3p)

7. Let $\{X_t\}$ be an $MA(q)$ process with $b_k = (-1)^k$ and $\sigma_\epsilon = 1$. Observations of $\{X_t\}$ are going to be made and, based on these observations, the ef m_X will be estimated. Assuming that q is odd, calculate approximately how many observations that are needed to have $V(m_n^*) < \frac{1}{4}$. (5p)

GOOD LUCK!