Exam for Random Processes, 5 points/7.5 ECTS

August 24, 2005, 9.00 am – 1.00 pm

Max number of points: 30.
Halmstad University grading bounds: 12p ⇒ grade 3, 18p ⇒ grade 4, 24p ⇒ grade 5.
ECTS bounds: 12p ⇒ grade E, 15p ⇒ grade D, 18p ⇒ grade C, 21p ⇒ grade B, 24p ⇒ grade A.
Allowed aids: Sheet of formulae attached to the exam, calculator and Mathematics Handbook: Beta.
Examiner: Eric Järpe (035-16 76 53).

For each problem a *complete* solution should be given. All solutions should be thoroughly presented. Each solution should start at the top of a new sheet of paper. Only one solution a sheet. The proper solutions will be available on the internet at
`http://www.hh.se/staff/erja` → `Teaching` → `Random processes` → `050824: Solution`

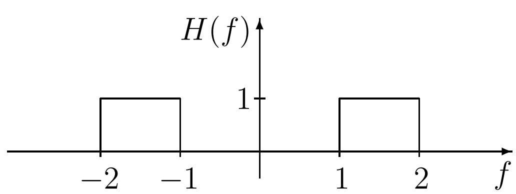
1. Assume that $\{X_t : t \in \mathbb{Z}\}$ is an $AR(2)$ process with parameters a_1 and a_2. Show that its spectral density is $R_X(f) = 1/(1 + a_1 e^{-i2\pi f} + a_2 e^{-i4\pi f})$. (4p)

2. Let $\{X_t : t \in \mathbb{R}\}$ be a Poisson process with intensity 1, let $Y_t = e^{3X_t}$ and let $Z_t = Y_t / Y_{t-2}$ for all $t \in \mathbb{R}$. Determine the
   (a) probability function of the variable Z_t. (3p)
   (b) expectation function of the process $\{Z_t\}$. (4p)

3. Let the random process $\{X_t : t \in \mathbb{R}\}$ in continuous time have cvf $r_X(\tau) = 2/(1 + 4\pi^2\tau^2)$ and let $Y_k = X_{2k}$ where $k = 0, \pm 1, \pm 2, \ldots$. (Thus the sequence $\{X_t\}$ is sampled at times $t = \ldots, -4, -2, 0, 2, 4, \ldots$) Calculate the spectral density, $R_Y(f)$, of the sampled process $\{Y_t\}$. (4p)

4. Let $\{W_t\}$ be a standard Wiener process and define $X_t = e^{-t/2} W_{e^t}$.
   (a) Determine the distribution of X_t. (4p)
   (b) Is the process $\{X_t\}$ strongly stationary? (3p)

5. A weakly stationary process $\{X_t\}$ with expectation 1 and spectral density function R_X is filtered in an ideal bandpass filter with transfer function $H(f)$ as plotted below.



   Express the mean effect (i.e. $E(Y_t^2)$) of $\{Y_t\}$ in terms of $R_X(f)$. (4p)

6. A stationary Gaussian process $\{X_t : t \in \mathbb{R}\}$ has expectation 0 and cvf $r_X(\tau) = \frac{1}{\pi} e^{-\pi\tau^2/2}$. Determine the crosscovariance function $r_{X,X'}(\tau) = C(X_t, X'_{t+\tau})$. (4p)

GOOD LUCK!