Exam for Random Processes, 5 points/7.5 ECTS

December 17, 2005, 9.00 am – 1.00 pm

Max number of points: 30.
Halmstad University grading bounds: 12p ⇒ grade 3, 18p⇒ grade 4, 24p ⇒ grade 5.
ECTS bounds: 12p ⇒ grade E, 15p ⇒ grade D, 18p ⇒ grade C, 21p ⇒ grade B, 24p ⇒ grade A.
Allowed aids: Summary of formulae attached to the exam, calculator and Math. Handbook: Beta.
Examiner: Eric Järpe (035-16 76 53).

For each problem a *complete* solution should be given. All solutions should be thoroughly presented. Each solution should start at the top of a new sheet of paper. Only one solution a sheet. The proper solutions will be available on the internet at
`http://www.hh.se/staff/erja` → `Teaching` → `Random processes` → `051217: Solution`

1. Show that the cvf of an $AR(p)$ process satisfies the Yule-Walker equations. (3p)

2. Let $\{N_t : t \in \mathbb{R}^+\}$ be a Poisson process with parameter 2. Calculate
   (a) $V(N_2 + N_3)$. (3p)
   (b) $E((-1)^{N_t})$. (4p)

3. Let $\{X_t : t \in \mathbb{R}\}$ be a weakly stationary process with spectral density function $R_X(f) = \delta_{-1}(f) + \delta_1(f)$. Calculate $C(X_t, X_{t+0.5})$. (3p)

4. Determine the cvf, $r_X(\tau)$, of an $MA(3)$ process with coefficients c_0, c_1, c_2 and c_3 where $c_0 = 2c_1 = 4c_2 = 8c_3 = 1$ and with $\sigma_\epsilon^2 = 64$. (4p)

5. Let $\{X_t : t \in \mathbb{R}\}$ be a Gaussian process with ef $m_X = 0$ and cvf $r_X(\tau) = 2e^{-|\tau|}$.
   (a) Calculate $P(X_t > 2)$. (3p)
   (b) Calculate $P(\int_{-1}^{1} X_t \, dt > 2)$. (3p)

6. Let $\{X_t : t \in \mathbb{R}\}$ be shot noise wiht intensity $\lambda = 30$ and impulse function $g(t) = \begin{cases} x^2 & \text{when } 0 < x < 1 \\ 0 & \text{o.w.} \end{cases}$. Determine the cvf of $\{X_t\}$. (3p)

7. Let $\{X_t\}$ be a weakly stationary process with cvf $r_X(\tau) = e^{-\tau^2}$. Determine approximately how many observations $X_1, X_2, \ldots, X_n$ are needed for the variance of the expectation estimator $m_n^* = \frac{1}{n}\sum_{t=1}^{n} X_t$ to be smaller than 0.1. (4p)

GOOD LUCK!