Exam for Random Processes, 5 points/7.5 ECTS

January 13, 2006, 9.00 am – 1.00 pm

Max number of points: 30.
Halmstad University grading bounds: 12p ⇒ grade 3, 18p⇒ grade 4, 24p ⇒ grade 5.
ECTS bounds: 12p ⇒ grade E, 15p ⇒ grade D, 18p ⇒ grade C, 21p ⇒ grade B, 24p ⇒ grade A.
Allowed aids: Summary of formulae attached to the exam, calculator and Math. Handbook: Beta.
Examiner: Eric Järpe (035-16 76 53).

For each problem a *complete* solution should be given. All solutions should be thoroughly presented. Each solution should start at the top of a new sheet of paper. Only one solution a sheet. The proper solutions will be available on the internet at
`http://www.hh.se/staff/erja` →`Teaching` →`Random processes` →`060113: Solution`

1. Show that if g is the inverse Fourier transform of G and $H = g$, then $h(\tau)$ (the inverse Fourier transform of H) is $G(-\tau)$. (3p)

2. An elevator has visited the ground floor N_t times by time t (in hours from 8 o'clock in the morning), where $\{N_t : t \in \mathbb{R}^+\}$ is a Poisson process with intensity 3. What is the probability that $N_t + 5 < N_{t+2}$? (3p)

3. The weakly stationary Gaussian process $\{X_t : t \in \mathbb{Z}\}$ has cvf

$$r_X(\tau) = \begin{cases} 2 & \text{if } \tau = 0 \\ -1 & \text{if } |\tau| = 1 \\ 0 & \text{otherwise} \end{cases}$$

   (a) Calculate $P(X_t - X_{t+1} < 1)$ (3p)
   (b) In what frequency is the spectral density of $\{X_t\}$ maximal? (3p)

4. Suppose the weakly stationary process $\{X_t\}$ has spectral density $R_X(f) = e^{-\sqrt{|f|}}$, and that $\{X_t\}$ is filtered with impulse response $h(t) = 1$ if $0 \leq t \leq 1$ and $h(t) = 0$ otherwise. What is the spectral density of the filtered process? (4p)

5. Let $\{X_t\}$ be a positively correlated $AR(1)$ process with $\sigma_\epsilon^2 = \frac{1}{4}$ and $\sigma_X^2 = 1$.
   (a) Determine the value of the AR parameter a_1. (3p)
   (b) Calculate the cvf $r_X(\tau)$ of the process $\{X_t\}$. (3p)

6. Suppose the process $\{X_t : t \in \mathbb{R}\}$ which has spectral density function $R_X(f) = e^{-|f|}$ is sampled at time-points $t = 0, \pm\frac{1}{3}, \pm\frac{2}{3}, \pm 1, \pm\frac{4}{3}, \ldots$ What is the spectral density of the sampled signal. (4p)

7. Let $Y_t = \sqrt{e^{W_t^2}}$ where $\{W_t\}$ is a standard Wiener process. Calculate the density function of Y_t. (4p)

GOOD LUCK!