Exam for Random Processes, 5 points/7.5 ECTS

August 18, 2006, 9.00 am – 1.00 pm

Max number of points: 30.
Halmstad University grading bounds: 12p ⇒ grade 3, 18p⇒ grade 4, 24p ⇒ grade 5.
ECTS bounds: 12p ⇒ grade E, 15p ⇒ grade D, 18p ⇒ grade C, 21p ⇒ grade B, 24p ⇒ grade A.
Allowed aids: Summary of formulae attached to the exam, calculator and Math. Handbook: Beta.
Examiner: Eric Järpe (035-16 76 53, 0702-822 844).

For each problem a *complete* solution should be given. All solutions should be thoroughly presented. Each solution should start at the top of a new sheet of paper. Only one solution a sheet. The proper solutions will be available on the internet at
`http://www.hh.se/staff/erja` → `Teaching` → `Random processes` → `060813: Solution`

1. Assume $\{X_t\}$ is a weakly stationary process which is filtered with a linear filter with transfer function H resulting in the output $\{Y_t\}$. Show that $V(Y_t) = \int_{\mathbb{R}} |H(f)|^2 R_X(f)\,df$. (4p)

2. Suppose that $\{X_t\}$ is a Poisson process with intensity 4 and let $Y_t = \frac{1}{2}(X_t - 4t)$. Calculate
   (a) $C(X_t, X_{t+1})$. (3p)
   (b) $P(Y_t > Y_{t+1})$. (4p)

3. Can
$$R(f) = \begin{cases} 1 + \cos(\pi f) & \text{if } -1 \le f < 1 \\ 0 & \text{otherwise} \end{cases}$$
   be the spectral density function of a weakly stationary process in continuous time? Motivate your answer. (3p)

4. Assume that $\{\epsilon_t\}$ is normally distributed white noise with $\sigma_\epsilon^2 = 1$ and that the process $\{X_t : t \in \mathbb{Z}\}$ is defined by $X_t = \epsilon_t \epsilon_{t-1}$ for all t.
   (a) Is the white noise process, $\{\epsilon_t\}$, strongly stationary? (2p)
   (b) Show that $\{X_t\}$ is a sequence of uncorrelated variables. (4p)

5. An $AR(1)$ process, $\{X_t\}$, is realised at consecutive timepoints:
$$x_1 = -0.54 \quad x_2 = 0.22 \quad x_3 = 0.07 \quad x_4 = -2.20 \quad x_5 = 0.75 \quad x_6 = -1.01 \quad x_7 = 0.58$$
   By using these observations, estimate the covariance between X_t and X_{t-1}. (4p)

6. Let $\{X_t : t \in \mathbb{R}\}$ be a weakly stationary process with $m_X = 1$ and $r_X(\tau) = \max(0, 1 - |\tau|)$. Further let the process $\{Y_t\}$ be defined by $Y_t = \int_0^t X_s\,ds$. Determine the the spectral density, $R_Y(f)$, of the output signal. (4p)

GOOD LUCK!