Exam for Random Processes, 5 points/7.5 ECTS

April 21, 2006, 9.00 am – 1.00 pm

Max number of points: 30.
Halmstad University grading bounds: 12p ⇒ grade 3, 18p⇒ grade 4, 24p ⇒ grade 5.
ECTS bounds: 12p ⇒ grade E, 15p ⇒ grade D, 18p ⇒ grade C, 21p ⇒ grade B, 24p ⇒ grade A.
Allowed aids: Summary of formulae attached to the exam, calculator and Math. Handbook: Beta.
Examiner: Eric Järpe (035-16 76 53).

For each problem a *complete* solution should be given. All solutions should be thoroughly presented. Each solution should start at the top of a new sheet of paper. Only one solution a sheet. The proper solutions will be available on the internet at
`http://www.hh.se/staff/erja` → `Teaching` → `Random processes` → `060421: Solution`

1. Show that if $\{X_t\}$ is weakly stationary, H is a transfer function for a linear filter, $\{Y_t\}$ is the filtered process and R_X and R_Y are the respective spectral densities. then $R_Y(f) = |H(f)|^2 R_X(f)$. (4p)

2. For a stationary process in continuous time, $\{X_t\}$, and its derivative in squared mean, $\{X'_t\}$, show that $C(X_t, X'_t) = 0$. (3p)

3. Can $r(\tau) = \frac{1}{1+4\pi^2(1-\tau)^2} + \frac{1}{1+4\pi^2(1+\tau)^2}$ be cvf of a weakly stationary process? (3p)

4. Let the process $\{X_t\}$ be shot noise with intensity $\ln 2$ and impulse function

$$g(t) = \begin{cases} 2^{-t} & \text{for } t \geq 0 \\ 0 & \text{o.w.} \end{cases}$$

   Determine approximately the probability $P(X_t > 1 + X_{t-1})$. (4p)

5. The observations 1.2, 5.3, 1.7, 0.8, 3.1 of a strongly stationary process $\{X_t\}$ are made. Estimate the cvf of $\{X_t\}$ from this sample. (3p)

6. Let $\{X_t\}$ be an $AR(1)$ process with parameter $\frac{1}{2}$, i.e. $X_t = -0.5X_{t-1} + \epsilon_t$ for all $t \in \mathbb{Z}$ where $\{\epsilon_t\}$ is normally distributed white noise with variance $\sigma_\epsilon^2 = 3$.
   (a) Calculate $C(X_1, X_2)$. (4p)

   Let $\{Y_t\}$ be an $AR(1)$ process with parameter $-\frac{1}{5}$, i.e. $Y_t = 0.2Y_t + \eta_t$ where $\{\eta_t\}$ is normally distributed white noise with variance $\sigma_\eta^2 = 1$ independently of $\{\epsilon_t\}$.
   (b) Determine numbers $\alpha, \beta \in \mathbb{R}$ such that $\{Z_t\}$ is an $AR(1)$ process where $Z_t = \alpha X_t + \beta Y_{t-1}$ for all $t \in \mathbb{Z}$ and calculate the variance $\{Z_t\}$. (5p)

7. Let $\{N(t) : t \geq 0\}$ be a Poisson process and let the process $\{Y_t\}$ be defined by $Y_t = e^{-t}N(e^{2t}) - e^t$ for all $t \in \mathbb{R}^+$. Show that $\{Y_t\}$ is weakly stationary. (4p)

GOOD LUCK!