EXAM FOR STOCHASTIC MODELS IN DISCRETE TIME
3.75 ECTS

Master's program of Financial Mathematics
August 11, 2009, 9.00 – 13.00

Max number of points: 30.
Halmstad University grading bounds: 12p ⇒ grade 3, 18p⇒ grade 4, 24p ⇒ grade 5.
ECTS bounds: 12p ⇒ grade E, 15p ⇒ grade D, 18p ⇒ grade C, 21p ⇒ grade B, 24p ⇒ grade A.
Allowed aids: Summary of formulae attached to the exam, calculator and dictionary.
Examiner: Eric Järpe (035-16 76 53, 0702-822 844).

For each problem a *complete* solution should be given. All solutions should be thoroughly presented. Each solution should start at the top of a new sheet of paper. Only one solution a sheet.
The proper solutions will be available on the internet at `http://www.hh.se/staff/erja` → `Teaching` → `Financial Mathematics` → `Stochastic models` → `Previous exams`

1. Prove that if $\{X_n\}_0^N$ is a local martingale with respect to the filtration $\{\mathcal{F}_n\}_0^N$ such that $E(\min(0, X_N)) > -\infty$ and $E(\max(0, X_N)) < \infty$, then $\{X_n\}_0^N$ is a martingale with respect to $\{\mathcal{F}_n\}_0^N$. (4p)

2. In a financial market, each day n, the brokers buy for the amount X_n and sell for Y_n, where $X_n \in Poi(3)$ and $Y_n \in Poi(2)$ in millions of dollars and $\{X_n\}$ and $\{Y_n\}$ are indpependent between days and of each other. What is
   (a) the probability that the business volume (i.e. amount buy for and amount sell for added) exceeds \$5 million during one day? (3p)
   (b) approximately the probability that the business volume exceeds \$1.2 billion (i.e. \$1200 million) during one year (i.e. 250 days)? (5p)

3. Assume $\{X_t\}$ is an AR(2) process with $E(X_t) = 100$, $V(X_t) = 4$, $C(X_t, X_{t+1}) = 3$ and $C(X_t, X_{t+2}) = 2$. Determine the parameters a_0, a_1, a_2 and σ_ϵ^2. (4p)

4. An index, $\{X_n\}$, is described by a GARCH(1,1) process with $a_0 = 111$, $a_1 = 0.1$ and $a_2 = 0.5$. Calculate the
   (a) variance of X_n. (4p)
   (b) probability that the index exceeds 100 at time $n + 1$ given that it was 99 at time n, and that $\sigma_n = 50$. (5p)

5. Let $\{M_n\}$ be a martingale with respect to the filtration $\{\mathcal{F}_n\}$, where $\mathcal{F}_n = \sigma(M_1, M_2, \ldots, M_n)$. Show that $\{X_n\}$, defined by $X_n = e^{M_n}$, is a submartingale with respect to $\{\mathcal{F}_n\}$. (5p)

GOOD LUCK!