Written Exam in Mobile Intelligent Systems

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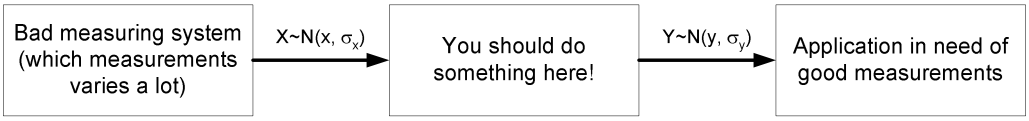
Assistant aids:	Writing tools, Calculator and an arbitrary book on formulas (e.g. Beta)
Location / Date:	Room R1122, Halmstad / 2004-05-25
Time limit:	13:30 – 17:30 => 4 hours
Answers:	All answers should be motivated.
Language:	Write your answers in either **Swedish** or **English** language.
Contact:	Ola Bengtsson, 035 – 167485 or 0733 – 121281
Parts:	**Part I**: Basic questions: Maximum points = 28 [11.0 – 16.5]p gives grade = 3 [17.0 – 22.0]p gives grade = 4 [22.5 – 28.0]p gives grade = 5 **Part II**: Deeper understanding of a scientific paper Maximum points = 18 [07.5 – 10.5]p gives grade = 3 [11.0 – 14.0]p gives grade = 4 [14.5 – 18.0]p gives grade = 5
Passing the exam / Final grade:	You should, to pass the exam, at least have the grade 3 on both individual parts, i.e. you have to pass both parts. The grade is then given as the average of the two individual grades.

Good luck,

/Ola.

Part I: Basic Questions

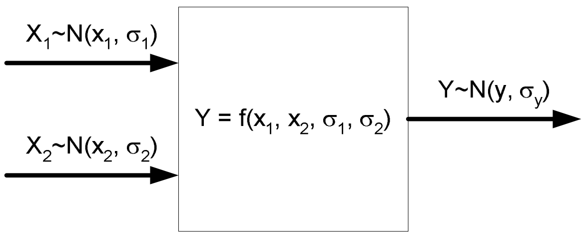
1.1) You have bought a bad (i.e. the measurements varies a lot) sensor for measuring ranges. The error of the sensor is assumed Gaussian with zero mean and known variance, σ_x^2, i.e. the error follows ~ $N(0, \sigma_x)$. See the below figure for an illustration.



A) What can you do to feed your applications with better measurements (a measurement with smaller variance) than you can get from your sensor system? (1p)

B) What should you do to get a standard deviation, σ_y, that is 10 times smaller than σ_x, i.e. how many measurements do you have to use and how should you use them? (2p)

1.2) Assume you have two independent (both having errors that are Gaussian distributed) measuring systems, measuring X_1 and X_2 and having the variances σ_1^2 and σ_2^2. See the below figure for an illustration.



A) Derive the expression for the linear combination Y = f(X_1, X_2) that gives you the smallest variance of Y. (4p)

B) In question 1.2A you should, hopefully, come up with the following equation:

$$Y = \frac{\sigma_2^2}{\sigma_1^2 + \sigma_2^2} X_1 + \frac{\sigma_1^2}{\sigma_1^2 + \sigma_2^2} X_2$$

What happens to Y if $\sigma_1^2 = \sigma_2^2$ and if $\sigma_1^2 << \sigma_2^2$? (2p)

C) Derive an expression for the variance of Y. (2p)

D) It is crucial that the errors of X_1 and X_2 are well represented by σ_1^2 and σ_2^2 - why? What happens if either of the two measuring systems overestimates its error, i.e. the error is actually a lot bigger than the variance? (1p)

1.3) Assume you have a position $(x_R \quad y_R \quad \theta_R)^T$ estimate of a mobile robot and that you have the positions of different beacons stored in a map $(x_B \quad y_B)^T$. This means that you can predict the angle to any of the beacons in the map from any given position (i.e. from the current position of the robot) by the following equation:

$$\hat{\varphi} = \tan^{-1}\left(\frac{y_R - y_B}{x_R - x_B}\right) - \theta_R$$

A) Assume that the robots position and the position of the beacon are uncertain parameters (Gaussian distributed with known co-variance matrix) – derive the uncertainty of $\hat{\varphi}$ (It is assumed that the uncertainty of $\hat{\varphi}$ should be Gaussian distributed). (3p)

1.4) When using an observation the Kalman filter has a cyclic behavior, i.e. consists of: a prediction, a matching and an updating step. If we assume that the sensor system observes e.g. vertical lines in the environment, briefly explain what happens (the main purpose) in:

A) the prediction step, (3p)
B) the matching step and (2p)
C) the update step. (1p)

1.5) A validation gate (which often is based on the Mahalanobis distance) is often used in the matching step (question 1.4B).

A) What is the purpose of this validation gate? (1p)
B) What happens to your localization system (as the one explained in question 1.4) if you chose a too <u>small</u> validation gate? (1p)
C) What happens to your localization system (as the one explained in question 1.4) if you chose a too <u>large</u> validation gate? (1p)

1.6) The methods based on position probability grids have both advantages, e.g. can localize the robot globally, and disadvantages.

A. What is represented by the position probability grid? (1p)
B. Give three examples of problems with the Markov localization that are solved by the particle filter method. (3p)

Part II: Deeper understanding of a Scientific Paper

The following questions are all on the attached paper, 'Blanche – An Experiment in Guidance and Navigation of an Autonomous Robot Vehicle' by Ingemar Cox, 1991.

A. What does the equation in Section IV:B – Sensor Data tells you? Describe the parts (x, y, C, R, r and α) by a schematic drawing of the robot and its sensor system. (4p)

B. Summarize the three steps of the iterative algorithm, referred to as the Cox matching algorithm, described in the paper. What are the main ideas and results of the different steps? What error (distance) is minimized in the final step (illustrate this with a figure)? (6p)

C. When is the algorithm terminated, i.e. when do the iterations stop? (1p)

D. In the end of the description of the algorithm the author mentions something about rejecting outliers, what does this mean? Why is this important? What would happen if the outliers where not rejected? Also suggest one way how you can reject such outliers. (3p)

E. The algorithm also, together with a result, calculates a co-variance matrix, which represents the error of the match result, i.e. how reliable the result is in x, y and θ. How would this co-variance matrix look like if the matches where done in the environments shown in the three figures below. (The <u>left</u> figure shows a robot standing in the middle of a long corridor and where the ends of the corridor are out of sight for the sensor. The <u>middle</u> figure shows a robot standing in the middle of a square environment and with all walls visible to the sensor. The <u>right</u> figure shows a robot standing in the middle of a circular environment and where all the walls are visible to the sensor.) (4p)