Theory for Stochastic models in discrete time, 2.5 p

One should be able to phrase and prove the following results:

Thm *(pp 98–99)*

Martingale equivalences

If $X = \{X_n, \mathcal{F}_n : n \geq 0\}$ is a stochastic sequence with $E(|X_0|) < \infty$, then
$X \in \mathcal{M}_{loc} \quad \Leftrightarrow \quad X \in G\mathcal{M} \quad \Leftrightarrow \quad X \in \mathcal{M}T$

Lemma *(p 100)*

Conditions for a local martingale to be a martingale

(*i*) If $\{X_n, \mathcal{F}_n : n \geq 0\}$ is a local martingale such that $E(X_n^-) < \infty$ or $E(X_n^+) < \infty$, then $\{X_n, \mathcal{F}_n : n \geq 0\}$ is a martingale.

(*ii*) If $\{X_n, \mathcal{F}_n : 0 \leq n \leq N\}$ is a local martingale such that $E(X_N^-) < \infty$ or $E(X_N^+) < \infty$, then $\{X_n, \mathcal{F}_n : 0 \leq n \leq N\}$ is a martingale.

where $X_n^+ = \max(0, X_n)$ and $X_n^- = -\min(0, X_n)$

Obs *(pp 388–389)*

Balance condition for selffinancing strategies in case of dividends

If $\pi \in SF$ in a (B, S)-market with dividends $\{D_n\}$, then

$$\left\{ \begin{array}{c} X_n^\pi \; = \; \beta_n B_n + \gamma_n (S_n + D_n) \\ B_{n-1} \Delta\beta_n + (S_{n-1} + D_{n-1}) \Delta\gamma_n = 0 \end{array} \right\} \Longleftrightarrow \; \Delta X_n^\pi \; = \; \beta_n \Delta B_n + \gamma_n (\Delta S_n + \Delta D_n)$$

for all $n \in \mathbb{Z}^+$

Obs *(pp 396–397)*

Oportunities for arbitrage

(*i*) If a contract is sold for a price greater than $\mathbb{C}^*$ then there exists an opportunity for arbitrage for the seller.

(*ii*) If a contract is bought for a price less than $\mathbb{C}_*$ then there exists an opportunity for arbitrage for the buyer.

where $\mathbb{C}^*$ is the upper price and $\mathbb{C}_*$ is the lower price of hedging against some non-negative $\mathcal{F}_N$-measurable function f_N.

Thm *(pp 413, 417)*

Martingale criterion of the absence of arbitrage

A (B, S)-market with bank account $B = \{B_n : n = 1, 2, \ldots, N\}$ and finitely many assets $S = \{(S_n^1, S_n^2, \ldots, S_n^d) : n = 1, 2, \ldots, N\}$ is defined on the filtered probablility space $(\Omega, \mathcal{F}, \{\mathcal{F}_n\}, P)$. Then we have that
$\{$The (B, S)-market is arbitrage-free$\} \; \Longleftarrow \; \exists\, \tilde{P} \in \mathcal{P}(P) : \{\frac{S_n}{B_n}\}$ is a $\tilde{P}$-martingale
i.e. the existence of such martingale measure is a *sufficient condition* for the market to be arbitrage-free