Exam for Stochastic models in Discrete time 2.5 points/3.75 ECTS

Master's program of Financial Mathematics
May 4, 2007, 9.00 – 13.00

Max number of points: 30.
Halmstad University grading bounds: 12p ⇒ grade 3, 18p⇒ grade 4, 24p ⇒ grade 5.
ECTS bounds: 12p ⇒ grade E, 15p ⇒ grade D, 18p ⇒ grade C, 21p ⇒ grade B, 24p ⇒ grade A.
Allowed aids: Summary of formulae attached to the exam, calculator and dictionary.
Examiner: Eric Järpe (035-16 76 53, 0702-822 844).

For each problem a *complete* solution should be given. All solutions should be thoroughly presented. Each solution should start at the top of a new sheet of paper. Only one solution a sheet.

The proper solutions will be available on the internet at
`http://www.hh.se/staff/erja` → `Teaching` → `Financial Mathematics` → `Stochastic models` → `Previous exams` → `070504: Solution`

1. Show that if $\mathbb{C}^*$ is the upper price of hedging against some non-negative $\mathcal{F}_N$-measurable function f_N and a contract is sold for a price greater than $\mathbb{C}^*$, then there exists an opportunity for arbitrage for the seller. (5p)

2. Suppose $\{h_t : t \in \mathbb{Z}^+\}$ is an $AR(1)$ process with parameters $a_1 = 0.78$ and white noise variance $\sigma_\epsilon^2 = 1$. Then calculate
   (a) $V(h_t)$, (3p)
   (b) $P(h_t + h_{t-1} \leq 1)$. (4p)

3. Suppose that $\{h_t : t \in \mathbb{Z}^+\}$ is a $GARCH(1,1)$ model.
   (a) Calculate the first moment of $\{h_t\}$. (3p)
   (b) Caclulate the second moment of $\{h_t\}$. (4p)
   (c) Assuming $a_1 > 0$, show that $\{h_t\}$ is leptokurtic. (6p)

4. Is the process $\{X_n : n = 0, 1, 2, \ldots\}$ a martingale, a submartingale or a supermartingale with respect to the σ-algebra $\mathcal{F}_n = \{\sigma(Y_0, Y_1, \ldots, Y_n)\}$ where $X_n = \max(-1, Y_n)$ and $\{Y_n\}$ is a random walk with $Y_0 = 0$ and $P(Y_n - Y_{n-1} = 1) = = P(Y_n - Y_{n-1} = -2) = 0.5$ for all $n = 1, 2, 3, \ldots$? (5p)

GOOD LUCK!