Exercise Exam for Stochastic models in Discrete time
2.5 points/3.75 ECTS

Master's program of Financial Mathematics
October 20, 2006, 9.00 – 13.00

Max number of points: 30.
Halmstad University grading bounds: 12p ⇒ grade 3, 18p⇒ grade 4, 24p ⇒ grade 5.
ECTS bounds: 12p ⇒ grade E, 15p ⇒ grade D, 18p ⇒ grade C, 21p ⇒ grade B, 24p ⇒ grade A.
Allowed aids: Summary of formulae attached to the exam, calculator and dictionary.
Examiner: Eric Järpe (035-16 76 53, 0702-822 844).

For each problem a *complete* solution should be given. All solutions should be thoroughly presented. Each solution should start at the top of a new sheet of paper. Only one solution a sheet. The proper solutions will be available on the internet at
`http://www.hh.se/staff/erja` → `Teaching` → `Financial Mathematics` → `Stochastic models` → `Previous exams` → `061020 exercise: Solution`

1. Show the equivalence for strategies $\pi \in SF$ in a (B, S)-market with dividends

$$\left\{ \begin{array}{c} X_n^\pi = \beta_n B_n + \gamma_n(S_n + D_n) \\ B_{n-1}\Delta\beta_n + (S_{n-1} + D_{n-1})\Delta\gamma_n = 0 \end{array} \right\} \iff \Delta X_n^\pi = \beta_n \Delta B_n + \gamma_n(\Delta S_n + \Delta D_n)$$

   for all $n \in \mathbb{Z}^+$. (3p)

2. Calculate the second moment of the stationary HARCH(2) process. (3p)

3. Let $\{X_n : n \in \mathbb{Z}^+\}$ be a random walk white noise increments $\{\epsilon_n\}$.
   (a) Is the process $\{Y_n\}$ defined by $Y_n = \frac{1}{\sqrt{n}} X_n$ weakly stationary? (4p)
   (b) Show that $E(X_n) = 0$ but $\forall\, C \in \mathbb{R} : \lim_{n\to\infty} P(|X_n| > C) = 1$. (5p)

4. Consider the following game with a single dice: in round n we throw a fair dice and if the dice shows
   - 6, then we get 6 SEK
   - 1, 2, 3 or 4, then we must pay that amount SEK
   - 5, then we throw the dice again and then if the dice shows
     - 5 or 6, then we get twice that amount SEK
     - 1, 2, 3 or 4 then we must pay that amount SEK

   Determine whether this game is a martingale, a supermartingale or a submartingale for us. (3p)

5. Let $\{X_t\}$ be a stationary $AR(2)$ process with coefficients $a_1 = 0$ and $a_2 = a \neq 0$.
   (a) Calculate the value of σ_ϵ^2 if $a = \frac{1}{2}$ and $D(X_t) = 1$. (3p)
   (b) Determine the covariance function of $\{X_t\}$. (4p)
   (c) Derive the maximum likelihood estimator of a. (5p)

GOOD LUCK!