Exam for Stochastic models in Discrete time
2.5 points/3.75 ECTS

Master's program of Financial Mathematics
October 25, 2006, 9.00 – 13.00

Max number of points: 30.
Halmstad University grading bounds: 12p ⇒ grade 3, 18p⇒ grade 4, 24p ⇒ grade 5.
ECTS bounds: 12p ⇒ grade E, 15p ⇒ grade D, 18p ⇒ grade C, 21p ⇒ grade B, 24p ⇒ grade A.
Allowed aids: Summary of formulae attached to the exam, calculator and dictionary.
Examiner: Eric Järpe (035-16 76 53, 0702-822 844).

For each problem a *complete* solution should be given. All solutions should be thoroughly presented. Each solution should start at the top of a new sheet of paper. Only one solution a sheet.

The proper solutions will be available on the internet at
`http://www.hh.se/staff/erja` → `Teaching` → `Financial Mathematics` → `Stochastic models` → `Previous exams` → `061025: Solution`

1. Formulate and prove the necessity part of the Martingale Criterion of the Absence of Arbitrage. (6p)

2. Let $X_n = \sum_{k=k_0}^{n} c_k \epsilon_k$ where $c_k \in \mathbb{R}$ for all k and $\{\epsilon_k\}$ is white noise. Then what kind of process is $\{X_n\}$ and is it weakly stationary if
   (a) $k_0 = 0$ and $c_k = 1$ for all k? (4p)
   (b) $k_0 = n - 2$ and $c_k = 2^{k-n}$? (4p)

3. Let $\{X_t\}$ be a stationary $ARCH(1)$ process.
   (a) Calculate the variance of X_t. (4p)
   (b) Show that $\{X_t\}$ is leptokurtic. (4p)

4. Each day n an agent considers
   - either buying one asset for B_n SEK and, immediately, selling it for S_n SEK
   - or do nothing and wait for next day.
   The agent has the strategy to buy and sell at day n if $S_n > B_n$ and wait otherwise, which gives him the gain G_n of earnings up to day n. Assume that the sequences $\{B_n\}$ and $\{S_n\}$ are independent at all levels and that $B_n \in N(101, 1)$ and $S_n \in N(100, 1)$.
   (a) Show that the gain, $\{G_n\}$, from this strategy is a submartingale. (4p)
   (b) To come to terms with this arbitrage, a fee paid by the agent at each time n should be determined so that $\{G_n\}$ is a martingale. How large should this fee be? (4p)

GOOD LUCK!