Exam for Random Processes, 7.5 ECTS

April 18, 2009, 9.00 – 13.00

Max number of points: 30.
Halmstad University grading bounds: 12p ⇒ grade 3, 18p⇒ grade 4, 24p ⇒ grade 5.
Allowed aids: Summary of formulae attached to the exam, calculator and Math. Handbook: Beta.
Examiner: Eric Järpe (035-16 76 53, 0702-822 844).

For each problem a *complete* solution should be given. All solutions should be thoroughly presented. Each solution should start at the top of a new sheet of paper. Only one solution a sheet. The proper solutions will be available on the internet at
`http://dixon.hh.se/erja` → `Teaching` → `Random processes` → `Previous exams`

1. Show that a weakly stationary Gaussian process is strongly stationary. (4p)

2. Suppose $\{X_t : t \in \mathbb{Z}^+\}$ is defined by

$$X_t \;=\; aZ_t + Z_{t-1}$$

for all t where $\{Z_t\}$ is a Gaussian process with ef $m_Z(t) = 0$ and cvf $r_Z(\tau) = \max(0, 3 - |\tau|)$.

   (a) Determine the cvf $r_X(s,t)$ if $a = 1$. (3p)

   (b) For which values of a is $\{X_t\}$ strongly stationary? (3p)

   (c) Calculate $P(X_t > 1)$. (3p)

3. Let $\{N_t : t \in \mathbb{R}^+\}$ be a Poisson process with intensity 1.

   (a) Calculate $C(N_3, N_5)$. (3p)

   (b) Prove that the process $\{B_t : t \in \mathbb{R}^+\}$ defined by

$$B_t \;=\; \frac{N_{2^t} - 2^t}{\sqrt{2^t}} \text{ for all } t$$

   is weakly stationary. (4p)

4. Let $\{X_t : t \in \mathbb{R}\}$ be shot noise with intensity 0.01 and with magnitude of pulse according to the function

$$g(t) \;=\; \begin{cases} 0 & \text{for } t < 0 \\ e^{-t} & \text{for } t \geq 0 \end{cases}$$

   Calculate the cvf $r_X(\tau)$ of $\{X_t\}$. (4p)

5. Assume $\{X_t : t \in \mathbb{R}^+\}$ is a weakly stationary process with covariance function $r_X(\tau) = e^{2-2\tau^2}$ for all $\tau \in \mathbb{R}$. Determine the

   (a) spectral density function of $\{X_t\}$. (3p)

   (b) cross-covariance function of $\{X_t\}$ and the derivative process $\{X_t'\}$. (3p)

GOOD LUCK!