Exam for Random Processes, 7.5 ECTS

January 17, 2009, 9.00 – 13.00

Max number of points: 30.
Halmstad University grading bounds: 12p ⇒ grade 3, 18p⇒ grade 4, 24p ⇒ grade 5.
Allowed aids: Summary of formulae attached to the exam, calculator and Math. Handbook: Beta.
Examiner: Eric Järpe (035-16 76 53, 0702-822 844).

For each problem a *complete* solution should be given. All solutions should be thoroughly presented. Each solution should start at the top of a new sheet of paper. Only one solution a sheet. The proper solutions will be available on the internet at
`http://dixon.hh.se/erja` → `Teaching` → `Random processes` → `Previous exams`

1. Assume that $\{X_t\}$ is a weakly stationary random process which is differentiable in squared mean, and that r_X is the cvf of $\{X_t\}$ and $r_{X'}$ is the cvf of $\{X'_t\}$. Prove that $r_{X'}(\tau) = -r''_X(\tau)$. (4p)

2. Assume that $\{\epsilon_t : t \in \mathbb{Z}\}$ is white noise, and that the process $\{X_t : t \in \mathbb{Z}\}$ is defined by $X_t = \epsilon_t - 2\epsilon_{t-1} + \epsilon_{t-2}$ for all $t \in \mathbb{Z}$.
   (a) What is the process $\{X_t\}$ called? (3p)
   (b) Prove that $\{X_t\}$ is weakly stationary. (3p)

3. Consider the weakly stationary random process $\{X_t : t \in \mathbb{R}\}$ with expectation function $m(t) = 1$ and spectral density function $R(f) = f^2 e^{-|f|}$.
   (a) Determine the covariance function of $\{X_t\}$. (3p)

   Assume the random process $\{X_t\}$ is filtered with impulse response
   $h(t) = \delta_{-1}(t) + 2\delta_0(t) + \delta_1(t)$.
   (b) Calculate the spectral density of the filtered signal. (3p)

4. Calculate $C(X_1, X_2)$ if $\{X_t\}$ is shot noise with parameter 0.1 and pulse function
$$g(t) = \begin{cases} e^{-t} & \text{if } t > 0 \\ 0 & \text{if } t \le 0 \end{cases}$$
(3p)

5. Assume the random process $\{X_t : t \in \mathbb{R}\}$ has expectation function $m(t) = 1$ and covariance function $r(\tau) = e^{-\tau^2/2}$, $\tau \in \mathbb{R}$.
   (a) Determine the spectral density of $\{X_t\}$. (3p)
   (b) Assuming that $\{X_t\}$ is a Gaussian process, calculate the probability $P(X_t + X_{t+1} \le 1)$. (4p)
   (c) The process is to be sampled at times $0, d, 2d, 3d, \ldots$ Determine the sample interval, d, such that the risk of aliasing is smaller than 5%. (4p)

GOOD LUCK!