Exam for Random Processes, 7.5 ECTS

January 16, 2010, 9.00 – 13.00

Max number of points: 30.
Halmstad University grading bounds: 12p ⇒ grade 3, 18p⇒ grade 4, 24p ⇒ grade 5.
Allowed aids: Summary of formulae attached to the exam, calculator and Math. Handbook: Beta.
Examiner: Eric Järpe (035-16 76 53, 0702-822 844).

For each problem a *complete* solution should be given. All solutions should be thoroughly presented. Each solution should start at the top of a new sheet of paper. Only one solution a sheet. The proper solutions will be available on the internet at
`http://dixon.hh.se/erja` → `Teaching` → `Random processes` → `Previous exams`

1. Prove that $G = \mathcal{F}(g)$, $h(\tau) \equiv G(\tau) \;\Rightarrow\; H(f) \equiv g(-f)$ (where $H = \mathcal{F}(h)$ and $g = \mathcal{F}^{-1}(G)$). (4p)

2. Provet that if $\{X_t : t \in \mathbb{R}\}$ is differentiable, then $E(X_t') = 0$. (3p)

3. Let $\{X_t : t = 1,2,3,\ldots\}$ be defined by $X_t = \frac{N_t - \lambda t}{\sqrt{\lambda t}}$, $t = 1,2,3,\ldots$ where $\{N_t : t \in \mathbb{R}\}$ is a Poisson process with intensity λ.
   (a) Calculate $P(X_2 \le 1)$ if $\lambda = 3$. (3p)
   (b) Show that X_t is asymptotically standard normally distributed, i.e. give some arguments to why $\lim_{t\to\infty} P(X_t \le x) = \Phi(x)$. (4p)

4. Let $\{X_t : t \in \mathbb{Z}\}$ be an $AR(1)$ processs with $a_1 = -\frac{2}{3}$ and $\epsilon_t \in N(0, \frac{1}{2})$ (i.e. $V(\epsilon_t) = \frac{1}{2}$). Calculate $E(X_t)$ and $V(X_t)$. (3p)

5. The process $\{Y_t : t \in \mathbb{R}\}$ is weakly stationary with spectral density $R_Y(f) = |f|e^{-|f|}$. Now, when sampling Y_t one wants to choose the sampling distance such that the boundary value of the spectral density of the sampled process, $\{Z_t\}$, is not greater than 1, i.e. if one samples at $0, \pm d, \pm 2d, \pm 3d, \ldots$ the number d is to be chosen such that $R_Z(\frac{1}{2d}) \le 1$. What value should d have? (4p)

6. Let $\{\xi_t : t \in \mathbb{R}\}$ be weakly stationary process with cvf $r(\tau) = \frac{\cos\tau}{1+\tau^2}$.
   (a) Prove that $\{\xi_t\}$ is differentiable in squared mean. (3p)
   (b) Calculate $P(X_t^2 \le 2)$ assuming that $\{\xi_t\}$ is Gaussian with $m_\xi = 1$. (3p)
   (c) Assume the input $\{\xi_t\}$ is filtered using a filter with impulse response $h(t) = \frac{1}{\pi}\delta_{-\pi}(t) + \frac{1}{\pi}\delta_{\pi}(t)$. Determine the cvf of the output signal. (4p)

GOOD LUCK!