Exam for Random Processes, 7.5 ECTS

December 20, 2008, 9.00 am – 1.00 pm

Max number of points: 30.
Halmstad University grading bounds: 12p ⇒ grade 3, 18p⇒ grade 4, 24p ⇒ grade 5.
ECTS bounds: 12p ⇒ grade E, 15p ⇒ grade D, 18p ⇒ grade C, 21p ⇒ grade B, 24p ⇒ grade A.
Allowed aids: Summary of formulae attached to the exam, calculator and Math. Handbook: Beta.
Examiner: Eric Järpe (035-16 76 53, 0702-822 844).

For each problem a *complete* solution should be given. All solutions should be thoroughly presented. Each solution should start at the top of a new sheet of paper. Only one solution a sheet. The proper solutions will be available on the internet at
`http://dixon.hh.se/erja` → `Teaching` → `Random processes` → `Previous exams`

1. Assume $\boldsymbol{X} = (X_1, X_2, \ldots, X_n)$ is distributed according to the multivariate normal distribution. Show that if $C(X_i, X_j) = 0$ for all $i \neq j$, then X_i is independent of X_j for all $i \neq j$. (4p)

2. Suppose $\{X_t : t \in \mathbb{R}\}$ is a Poisson process with parameter $\lambda = 2$. Calculate
   (a) $E(X_t + 2X_{t+1})$, (3p)
   (b) the covariance function of $\{X_t\}$. (4p)

3. Let $\{X_t : t \in \mathbb{R}\}$ be a Gaussian process with expectation function $m(t) = 1$ and covariance function $r(\tau) = e^{-2|\tau|}$. Determine
   (a) $V(X_t)$, (2p)
   (b) $P(X_{t+1} - X_t > 1)$, (4p)
   (c) the spectral density function of $\{X_t\}$. (3p)

4. Suppose $\{X_t : t \in \mathbb{Z}\}$ is an $AR(1)$ process. Determine the value of the parameter σ_ϵ^2 if $V(X_t) = \sigma_X^2 = 1$ and $a_1 = -1/4$. (5p)

5. Assume that the process $\{X_t : t \in \mathbb{R}\}$ is filtered with the impulse response $\delta'(t)$. What is the output signal? (5p)
   Hint: For the delta function $\int_{\mathbb{R}} \delta'(u) f(u)\, du = \int_{\mathbb{R}} \delta(u) f'(u)\, du$. *Use this!*

GOOD LUCK!