Exam for Random Processes, 7.5 ECTS

April 17, 2010, 9.00 – 13.00

Max number of points: 30.
Halmstad University grading bounds: 12p ⇒ grade 3, 18p⇒ grade 4, 24p ⇒ grade 5.
Allowed aids: Summary of formulae attached to the exam, calculator and Math. Handbook: Beta.
Examiner: Eric Järpe (035-16 76 53, 0702-822 844).

For each problem a *complete* solution should be given. All solutions should be thoroughly presented. Each solution should start at the top of a new sheet of paper. Only one solution a sheet. The proper solutions will be available on the internet at
`http://dixon.hh.se/erja` → `Teaching` → `Random processes` → `Previous exams`

1. Prove the Yule Walker eqations. (4p)

2. Let $\{N_t\}$ be a Poisson process with intensity $\lambda = 0.001$. Determine
   (a) $Cov(N_{100}, N_{200})$. (3p)
   (b) approximately c such that $P(X_{999} < c) = 0.999$. (4p)

3. Assume $\{X_t : t \in \mathbb{R}\}$ is a Gaussian process with ef $m(t) = 1$ and cvf $r(\tau) = e^{1-\tau^2}$.
   (a) Calculate the spectral density of $\{X_t\}$. (3p)
   (b) Determine $P(X_t + X'_{t+1} \leq 5)$ where $\{X'_t\}$ is the derivative process. (4p)

4. Let $\{Y_t : t \in \mathbb{R}\}$ be shot noise with intensity $\lambda = 6$ and pulse function

$$g(t) = \begin{cases} 1 - t & \text{for } 0 < t < 1 \\ 0 & \text{otherwise} \end{cases}$$

   (a) Show that Y_5 and Y_6 are uncorrelated. (4p)
   (b) Calculate $E(\int_0^\infty e^{-t} Y_t \, dt)$. (3p)

5. Let the random process $\{X_t : t \in \mathbb{R}\}$ be defined by

$$X_t = A \cos(4\pi t + \phi)$$

   where $A > 0$ and ϕ are independent random variables, $V(A) < \infty$, $E(A^2) = a^2$ and $\phi \in R(0, 2\pi)$. Prove that $\{X_t\}$ is weakly stationary. (5p)

GOOD LUCK!