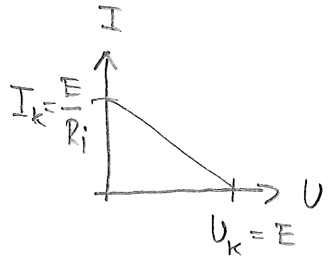
Lösningsförslag Omtenta 20110824

1)



$$U = E - R_i \cdot I$$

$$R_i = \frac{\Delta U}{\Delta I} = \{\text{ur tabell}\} = \frac{1{,}5023 - 1{,}4943}{16 \cdot 10^{-3} - 26 \cdot 10^{-3}}$$

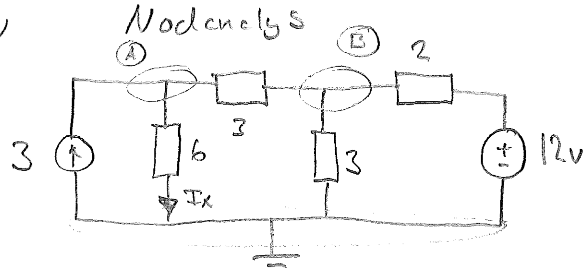
$$= \frac{0.008}{-10 \cdot 10^{-3}} = \underline{0{,}8\ \Omega}$$

obs! ej negativ resistens!

$$E = U + R_i \cdot I = \{\text{ur tabell}\} = 1{,}5023 + 0{,}8 \cdot 16 \cdot 10^{-3} = \underline{1{,}5151 \text{ volt}}$$

$$I_k = \frac{E}{R_i} = \frac{1{,}5151}{0{,}8} = \underline{1{,}89\ A}$$

2a) Nodanalys



Nod A KI "strömmar ut"

$$\frac{V_A - V_B}{3} + \frac{V_A}{6} - 3 = 0$$

$$\frac{1}{2} \cdot V_A - \frac{1}{3} \cdot V_B - 3 = 0$$

$$\frac{1}{2} \cdot V_A - 3 = \frac{1}{3} \cdot V_B \Rightarrow V_B = \frac{3}{2} V_A - 9 \quad = (1)$$

Nod B KI "strömmar ut"

$$\frac{V_B - V_A}{3} + \frac{V_B - 12}{2} + \frac{V_B}{3} = 0$$

$$\frac{V_B}{3} - \frac{V_A}{3} + \frac{V_B}{2} - 6 + \frac{V_B}{3} = 0 \Rightarrow \frac{2 \cdot 2}{3 \cdot 2} V_B + \frac{1 \cdot 3}{2 \cdot 3} V_B - \frac{V_A}{3} - 6 = 0$$

$$\Rightarrow \frac{7}{6} V_B - \frac{1}{3} V_A - 6 = 0 \quad = (2)$$

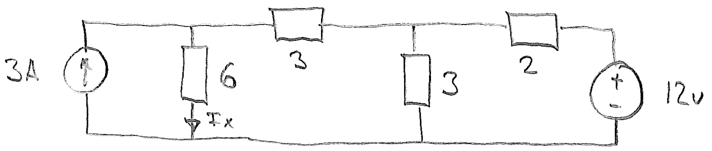
(1) och (2)

$$\frac{7}{6}\left(\frac{3}{2} V_A - 9\right) - \frac{V_A}{3} - 6 = 0 \Rightarrow \frac{21}{12} V_A - \frac{63}{6} - \frac{V_A \cdot 4}{3 \cdot 4} - \frac{6 \cdot 6}{6} = 0$$

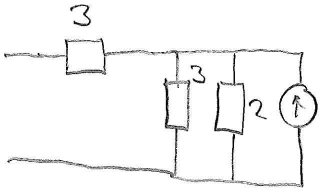
$$\Rightarrow \frac{17}{12} V_A - \frac{99 \cdot 2}{6 \cdot 2} = 0 \Rightarrow V_A = \frac{12 \cdot 198}{17 \cdot 12} = \frac{198}{17} = 11{,}647 \text{ volt}$$

$$I_x = \frac{V_A}{6} = \frac{11{,}647}{6} = \underline{1{,}94\ A}$$

2b) Succesiva 2pol ersättningar $E = R_0 \cdot J$ sökt I_x



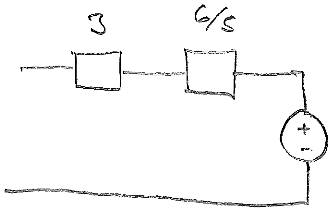
1)



$\frac{12}{2} = 6 \text{ A}$ $J = \frac{E}{R_0}$

$\frac{2 \cdot 3}{2+3} = \frac{6}{5}\ \Omega$

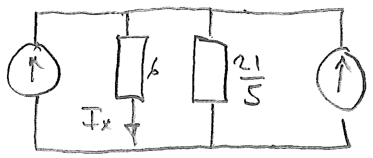
2)



$6 \cdot \frac{6}{5} = \frac{36}{5} \text{ V}$ $E = R_0 \cdot J$

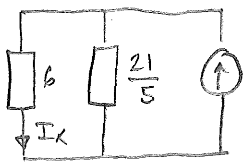
$3 + \frac{6}{5} = \frac{21}{5}\ \Omega$

3)



$\frac{\frac{36}{5}}{\frac{21}{5}} = \frac{36}{21}$ $J = \frac{E}{R_0}$

4)

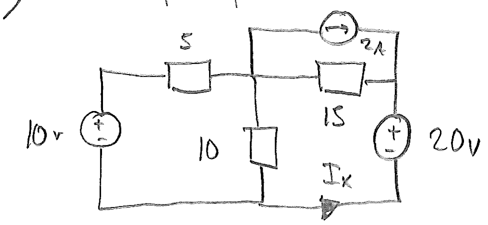


$J = \frac{36}{21} + 3 \cdot \frac{21}{21} = \frac{99}{21}$

Strömdelning gen

$I_x = \frac{\frac{21}{5}}{\frac{5 \cdot 6}{5} + \frac{21}{5}} \cdot \frac{99}{21} = \frac{21}{30+21} \cdot \frac{99}{21} = \frac{99}{51} = \underline{1{,}94 \text{ A}}$

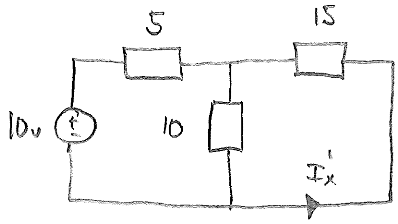
3) Superposition   sökt I_x



① ⇒ Avbrott

± ⇒ kortslutning

①



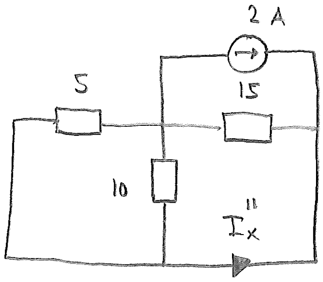
$$R_{tot} = 5 + 15 // 10 = 5 + \frac{15 \cdot 10}{25} = 11$$

$$I_{tot} = \frac{U}{R_{tot}} = \frac{10}{11} = 0,909$$

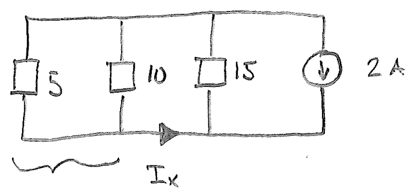
$$I'_x = \text{ström.del.} = \frac{10}{15+10} \cdot 0,909 = -0,3636$$

↑ obs! strömriktning

②



Lättare att se - riter om ⇒ ekv. schema

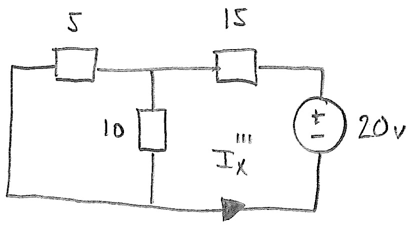


$$10 // 5 = \frac{10 \cdot 5}{10+5} = 3,333 \quad A$$

$$I''_x = \text{strömdelning} = \frac{15}{15 + 3,333} \cdot 2 = -1,6364 \text{ A}$$

↑ obs strömriktning

③



$$I'''_x = \frac{U}{R_{tot}} = \frac{20}{15 + \underbrace{5 // 10}_{3,333}} = 1,091 \text{ A}$$

$$I_x = I'_x + I''_x + I'''_x = \underline{-0,909 \text{ A}}$$

(4)

a)

$$u(t) = U_{DC} + |U| \sin(2\pi f \cdot t + \varphi)$$

Från figur:

Sinus:
- Amplitud: $|U| = 4V$
- Frekvens: $T = 20\ ms \Rightarrow f = \frac{1}{T} = \frac{1}{20 \cdot 10^{-3}} = 50 Hz$
- Fasvinkel: $\Delta t = 2{,}5 ms$

$$\varphi = \frac{\Delta t}{T} \cdot 360° = \frac{2.5}{20} \cdot 360° = 45°$$

(fördröjd signal $\Rightarrow -45°$)

$U_{DC} = 1V$

$$\Rightarrow \quad u(t) = 1 + 4 \sin(2\pi 50 t - 45°) \ V$$

b)

$$\overline{u(t)} = \frac{1}{T} \int_t^{t+T} u(t)\, dt$$

$$= \underbrace{\frac{1}{T} \int_t^{t+T} 1\, dt}_{=1} + \underbrace{\frac{1}{T} \int_t^{t+T} 4 \sin(2\pi 50 t - 45°)}_{=0}$$

$$\overline{u(t)} = 1 \ V \quad (= U_{DC})$$

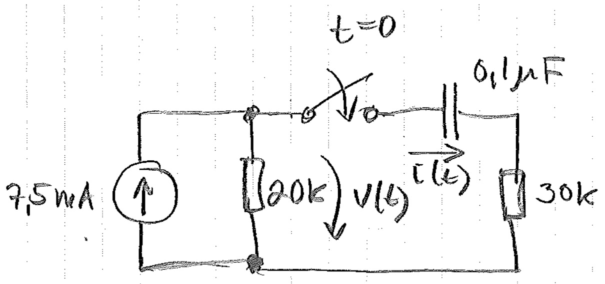
c)

AC-effektivvärdet

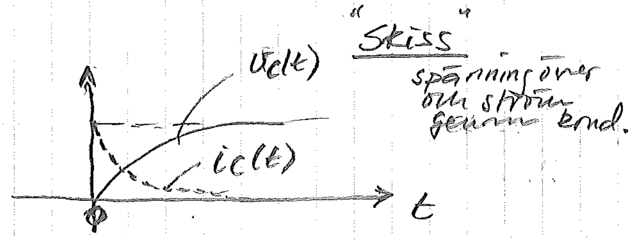
AC-signal: $u(t) - U_{DC} = 4 \sin(2\pi 50 \cdot t - 45°)$

U_e för sinus är $\frac{|U|}{\sqrt{2}} = \frac{4}{\sqrt{2}} \approx 2{,}83 \ V$

(5)



a) $i(t)$ då $t \geq 0+$



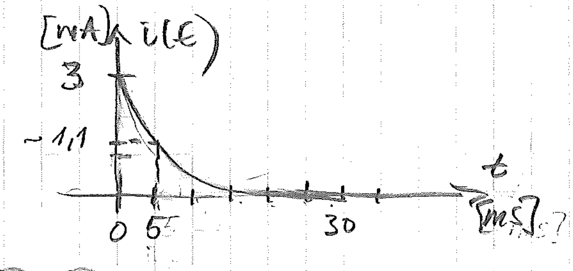
– Kondensatorn uppladdas.
Sp. tròg $\Rightarrow v_c(0) = 0$

strömdeln 20k/30k $\Rightarrow i(0) = 7.5 \cdot \frac{20}{(20+30)} = 7.5 \cdot \frac{20}{50} = 3\,mA$

– Uttryck för $i(t) = i(0) \cdot e^{-t/RC}$ (se skiss ovan!)

$RC = (20+30)k \cdot 0.1\mu = 50k \cdot 0.1\mu = 5 \cdot 10^{-3}\,s$
($R = R_{Th}$ sett från kond.)

$$\Rightarrow i(t) = 3 \cdot e^{-t/5 \cdot 10^{-3}}\,mA = 3 \cdot e^{-200t}\,mA$$



b) $v(t)$ då $t \geq 0+$; $v(t) = v_c(t) + 30k \cdot i(t)$

där:

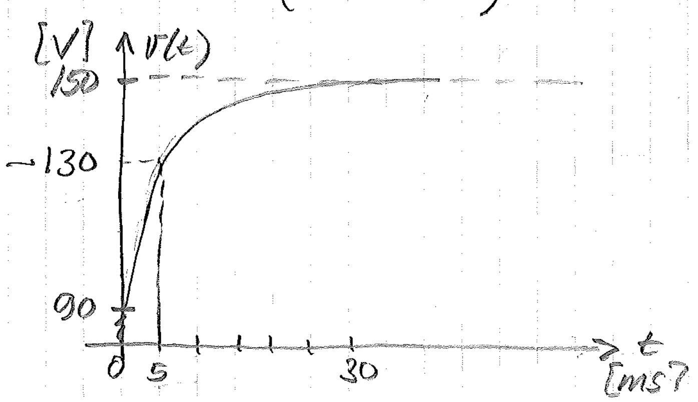
– $v_c(t) = E(1 - e^{-t/RC})$ uppladdning av kond

E är slutvärdet (då $i(t) = 0$ se "skiss"!)

dvs $E = v(t) = 20k \cdot 7.5m = 150\,V$
$RC = 5 \cdot 10^{-3}\,s$ (samma som tidigare!)

$$v(t) = v_c(t) + 30k \cdot i(t) = 150(1 - e^{-200t}) + 30k \cdot 3m\,e^{-200t}$$

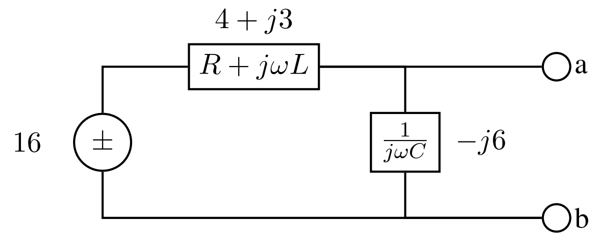
$$= 150(1 - e^{-200t}) + 90\,e^{-200t} = 150 - 60e^{-200t}\ V$$



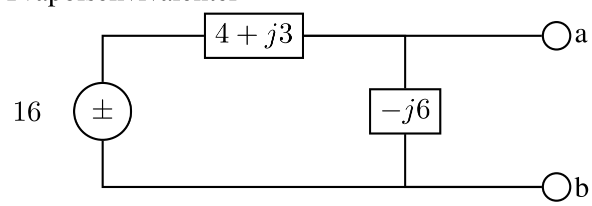
6. a) $Y_{AB} = Y_R + Y_L + Y_C = \frac{1}{R} + \frac{1}{j\omega L} + j\omega C = 2.63 + j6.17$ mS $= 6.70e^{j66.9^\circ}$ mS.

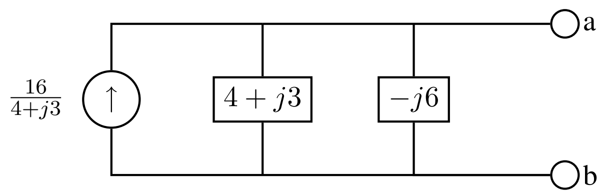
b) $I = EY_{AB} = 335e^{j66.9^\circ}$ mA, $\rightarrow i(t) = 335\sin(\omega t + 66.9^\circ)$ mA.

7. a) Komplext beräkningsschema

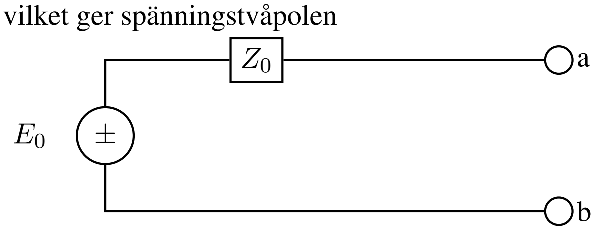


b) Tvåpolsekvivalenter





Impedansen för parallellkopplingen är $Z_0 = \frac{(4+j3)(-j6)}{4+j3-j6} = \frac{18-j24}{4-j3} = 5.76 - j1.68\,\Omega$ och spänningsekvivalenten är $E_0 = \frac{16}{4+j3} Z_0 = 19.2e^{-j53^\circ}$ V vilket ger spänningstvåpolen



c) Strömmen är $I_b = \frac{E_0}{Z_0 + R_b} = 2.4e^{j(\omega t - 41^\circ)}$ och effektutvecklingen är $P_R = RI_{be}^2 = 2(\frac{2.4}{\sqrt{2}})^2 = 2.4^2 = 5.8W$.

d) Välj $Z_b = Z_0^* \Rightarrow Z = Z_0 + Z_b = 2R_0$ vilket ger $I_{be} = \frac{E_{0e}}{|Z|} = \frac{E_{0e}}{2R_0}$ och effekten blir $P_b = R_0 I_{be}^2 = \frac{E_{0e}^2}{4R_0} = \frac{(19.2/\sqrt{2})^2}{4 \cdot 5.76} = 8W$.

8. a) Sammanställ aktiv och reaktiv effekt för varje apparat:

$P_1 = 35\ W$	$Q_1 = P_1 \tan(\arccos 0.85) = 21.7$ kVAr
$P_2 = 23.8\ W$	$Q_2 = 11.3$ kVAr
$P_3 = \lvert S_3\rvert \cos\varphi_3 = 13.6\ kW$	$Q_3 = \lvert S_3\rvert \sin(\arccos 0.76) = 11.63$ kVAr

$S = (P_1 + P_2 + P_3) + j(Q_1 + Q_2 + Q_3) = 72.4 + j44.6$ kVA, $|S| = 85$ kVA. Före kompensering:

$$I_e = \frac{|S|}{U_e} = 213\ A$$

Efter kompensering ($S = P$ och $Q = 0$):

$$I_e = \frac{P}{U_e} = 181\ A$$

b) Vid kompensering är $Q_1 + Q_2 + Q_3 + Q_C = 0$. Om man tar bort appart 3 får man kvar $P_1 + P_2$ och $Q_1 + Q_2 + Q_C = -Q_3$.

$$S = (P_1 + P_2) - jQ_3 = 58.8 - j11.6 \text{ kVA}$$
$$\varphi = \angle S = -11.2^\circ$$

Strömmen ligger 11.2° före spänningen och $\cos\varphi = 0.98$.