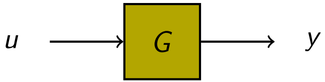
Lecture 3: Controller structures and implementation aspects

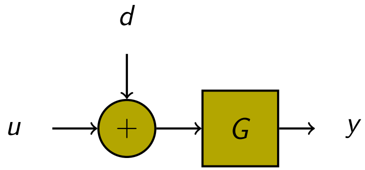
Outline

Block scheme interpretation

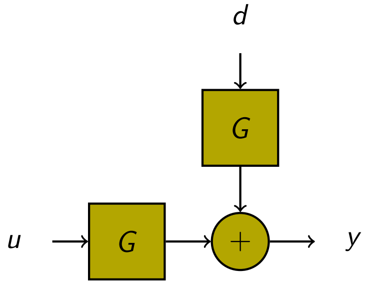


Read block scheme from right to left $y = Gu$

Block scheme distribution

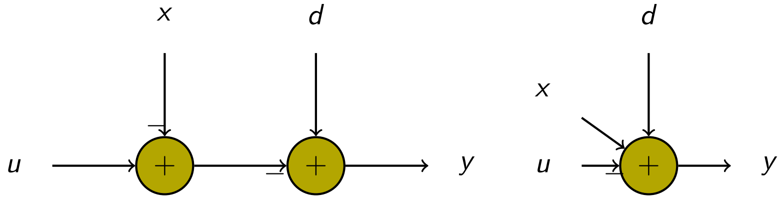


Put parenthesis at sum $y = G(u + d)$



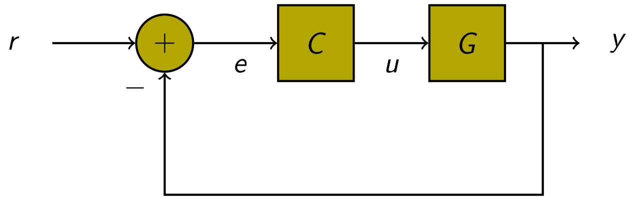
$y = Gu + Gd$

Block scheme summations



$$y = d - (u - x) = d - u + x$$

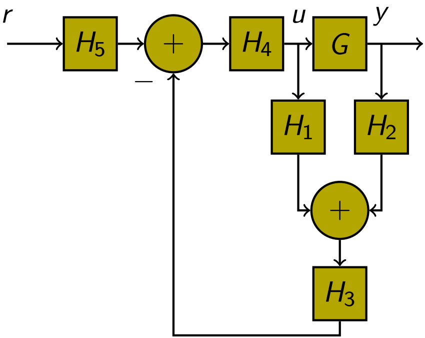
Block scheme with feedback

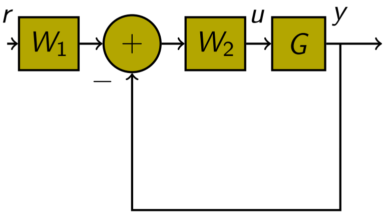


$$y = Gu = GCe = GC(r - y)$$
$$(1 + GC)y = GCr$$
$$y = \frac{GC}{1+GC} r$$

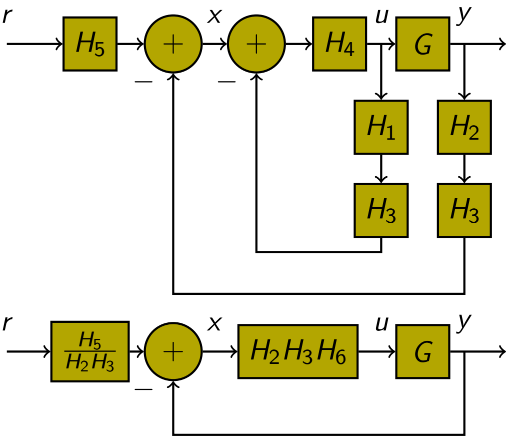
Example

Show equivalence between





Example: solution



Define inner loop H_6

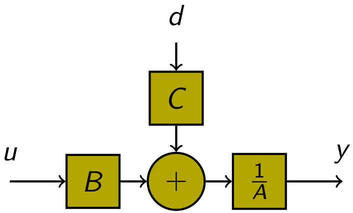
$$u = H_4(x - H_3 H_1 u)$$
$$\Rightarrow u = \frac{H_4}{1 + H_4 H_3 H_1} x$$
$$u = H_6 x$$

Consequently

$$\begin{cases} W_1 = \frac{H_5}{H_2 H_3} \\ W_2 = \frac{H_2 H_3 H_4}{1 + H_4 H_3 H_1} \end{cases}$$

Process structure

Control input u and disturbance d



After sum block

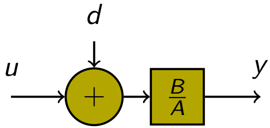
$$Ay = Bu + Cd$$

Output

$$y = \frac{B}{A}u + \frac{C}{A}d$$

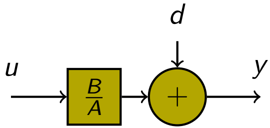
Input disturbance $C = B$

$y = \frac{B}{A}(u + d)$



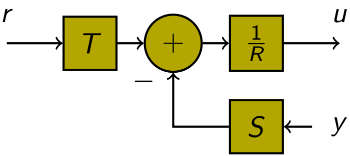
Output disturbance $C = A$

$y = \frac{B}{A}u + d$



Controller structure

Reference r and feedback y



After sum block

$$Ru = -Sy + Tr$$

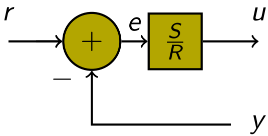
Control signal

$$u = -\frac{S}{R}y + \frac{T}{R}r$$

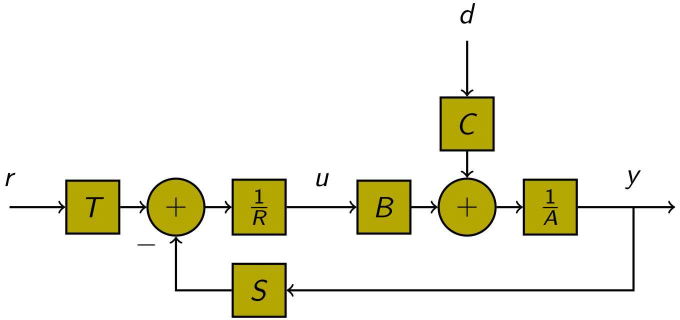
Error $e = r - y$

Special case: $T = S$

$$u = \frac{S}{R}e$$



Closed loop structure



Process

$$Ay = Bu + Cd$$

Controller

$$Ru = -Sy + Tr$$

Closed loop responses

From external signals r and d to internal signals y and u

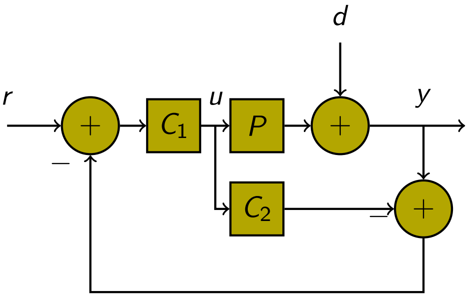
$$y = \frac{BT}{A_c} r + \frac{CR}{A_c} d$$
$$u = \frac{AT}{A_c} r - \frac{CS}{A_c} d$$

with closed-loop characteristic polynomial

$$A_c = AR + BS$$

Internal model control

Controller parameterized by C_1 and C_2

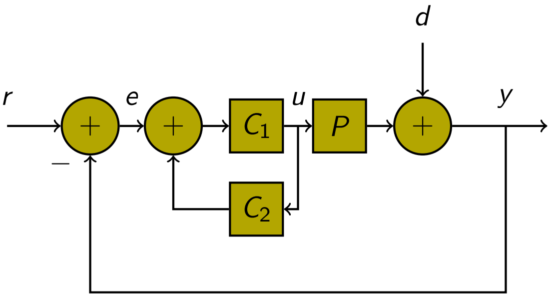


IMC tuning philosophy

- internal model $C_2 = P$ (feedback only $-d$)
- inverse model $C_1 = \frac{1}{P}$ (d eliminated and $y = r$)

Result $y = r$ (independent of d or is this too good to be true?)

IMC analysis

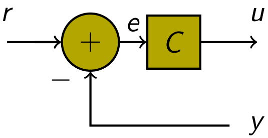


Internal loop $u = C_1(e + C_2 u) \Rightarrow u = \frac{C_1}{1 - C_1 C_2} e = He$

Outer loop $y = d + PH(r - y) \Rightarrow y = \frac{PH}{1+PH} r + \frac{1}{1+PH} d$

$$y = \frac{PC_1}{1 + C_1[P - C_2]} r + \frac{1 - C_1 C_2}{1 + C_1[P - C_2]} d$$

PID controller (book version)



Proportional, Integrating and Derivating (PID) controller

$$u(t) = P(t) + I(t) + D(t) = [Ke(t)] + [\frac{K}{T_i}\int_0^t e(s)ds] + KT_d\frac{d}{dt}e(t)]$$

In Laplace domain $U(s) = C(s)E(s)$

$$C(s) = K(1 + \frac{1}{T_i}\frac{1}{s} + T_d s)$$

PID controller (more realistic version)

$$u(t) = P(t) + I(t) + D(t)$$

ideally realized as

$$\begin{cases} P(t) = Ke(t) \\ \frac{d}{dt}I(t) = \frac{K}{T_i}e(t) \\ \frac{T_d}{N}\frac{dD(t)}{dt} + D(t) = -KT_d\frac{dy(t)}{dt} \end{cases}$$

but implemented by time-discrete approximation of derivatives

Time-discrete approximations of derivatives

Forward-difference (Euler) approximation

$$\frac{d}{dt}y(kh) \approx \frac{y(kh+h)-y(kh)}{h}$$

Derivative approximation by replacement

$$\frac{d}{dt} \to \frac{q-1}{h}$$

PI controller approximation ($s \to \frac{q-1}{h}$)

$$C_{PI}(s) = K(1+\tfrac{1}{T_i s}) \to K(1+\tfrac{h/T_i}{q-1}) =$$
$$K\frac{q-1+h/T_i}{q-1} = \frac{K+K(h/T_i-1)\mathrm{q}^{-1}}{1-\mathrm{q}^{-1}} = C_{PI}(\mathrm{q}^{-1})$$

Time-discrete approximations of derivatives

Backward-difference approximation

$$\frac{d}{dt}y(kh) \approx \frac{y(kh) - y(kh-h)}{h}$$

Derivative approximation by replacement

$$\frac{d}{dt} \rightarrow \frac{1 - \mathrm{q}^{-1}}{h}$$

PI controller approximation ($s \rightarrow \frac{1-\mathrm{q}^{-1}}{h}$)

$$C_{PI}(s) = K(1 + \tfrac{1}{T_i s}) \rightarrow K(1 + \tfrac{h/T_i}{1-\mathrm{q}^{-1}}) =$$
$$K\tfrac{1-\mathrm{q}^{-1}+h/T_i}{1-\mathrm{q}^{-1}} = \tfrac{K(1+h/T_i)-K\mathrm{q}^{-1}}{1-\mathrm{q}^{-1}} = C_{PI}(\mathrm{q}^{-1})$$

Time-discrete approximations of derivatives

Bilinear (Tustin) approximation by replacement

$$\frac{d}{dt} \rightarrow \frac{2}{h}\frac{1-\mathrm{q}^{-1}}{1+\mathrm{q}^{-1}}$$

PI controller approximation ($s \rightarrow \frac{2}{h}\frac{1-\mathrm{q}^{-1}}{1+\mathrm{q}^{-1}}$)

$$C_{PI}(s) = K(1+\frac{1}{T_i s}) \rightarrow K(1+\frac{h/(2T_i)(1+\mathrm{q}^{-1})}{1-\mathrm{q}^{-1}}) =$$
$$K\frac{1-\mathrm{q}^{-1}+h/(2T_i)(1+\mathrm{q}^{-1})}{1-\mathrm{q}^{-1}} = \frac{K(h/(2T_i)+1)+K(h/(2T_i)-1)\mathrm{q}^{-1}}{1-\mathrm{q}^{-1}} = C_{PI}(\mathrm{q}^{-1})$$

PI controller approximations

$$C_{PI}(s) = K(1 + \frac{1}{T_i s}) \rightarrow C_{PI}(\mathrm{q}^{-1}) = \frac{S(\mathrm{q}^{-1})}{R(\mathrm{q}^{-1})} = \frac{s_0 + s_1 \mathrm{q}^{-1}}{1 - \mathrm{q}^{-1}}$$

Forward-difference

$$\begin{cases} s_0 = K \\ s_1 = K(h/T_i - 1) \end{cases}$$

Backward-difference

$$\begin{cases} s_0 = K(1 + h/T_i) \\ s_1 = -K \end{cases}$$

Bilinear approximation

$$\begin{cases} s_0 = K(h/(2T_i) + 1) \\ s_1 = K(h/(2T_i) - 1) \end{cases}$$

Integrator windup

Control signal is limited

$$v(k) = sat[u(k)] = \begin{cases} u_{\min} & u(k) < u_{\min} \\ u(k) & u_{\min} \leq u(k) \leq u_{\max} \\ u_{\max} & u_{\max} < u(k) \end{cases}$$

Outside limitation feedback is broken!
Unstable controller is then running in open loop
Integrator windup: $u(k) = \sum_{n=0}^{k} e(n) \rightarrow \text{LARGE}$

Anti-windup

Designed controller

$$\begin{aligned} Ru &= -Sy + Tr \\ u(k) &= (1-R)u - Sy + Tr \\ &= \underbrace{-r_1 u(k-1) - r_2 u(k-2) - \dots}_{unbounded} - s_0 y(k) - \dots + t_0 r(k) + \dots \end{aligned}$$

Make controller stable during saturation ($v \neq u$)

$$\begin{aligned} u(k) &= (1-R){\color{red}v} - Sy + Tr \\ &= \underbrace{-r_1 {\color{red}v(k-1)} - r_2 {\color{red}v(k-2)} - \dots}_{bounded} - s_0 y(k) - \dots + t_0 r(k) + \dots \end{aligned}$$

Stable system with bounded $u(k)$

Ziegler-Nichols oscillation method

Use P-controller and measure

- gain of controller $K_{\max}$
- period time T_p of self-oscillation

when closed-loop is on the stability boundary

Tuning:

	K	T_i	T_d
P	$0.5K_{\max}$		
PI	$0.45K_{\max}$	$T_p/1.2$	
PID	$0.6K_{\max}$	$T_p/2$	$T_p/8$

Fine-tune manually