PI control of a tank process

In this exercise, **Sysquake** will be used to study the control design of a water filling tank process. Download the Sysquake file `Tank.sq` and open it in **Sysquake**. A tank process will then be displayed graphically. The input to the process is the water flow (equivalently the pump voltage) and the output is the level in the tank which is used for digital feedback control. If there were no outflow from the tank, the dynamics would be an 'integrator'. Here, however, there is a small outflow which makes the system stable. But since the outflow is much smaller than the inflow, it can be neglected in comparison. A crude approximation of the process dynamics can therefore be described by

$$y(k) = y(k-1) + u(k-1)$$

where y is the sampled level in the tank and u the input water flow (pump speed). In polynomial form, the system is

$$\mathsf{A}(\mathrm{q}^{-1})y(k) = \mathsf{B}(\mathrm{q}^{-1})u(k), \quad \mathsf{A}(\mathrm{q}^{-1}) = 1 - \mathrm{q}^{-1}, \quad \mathsf{B}(\mathrm{q}^{-1}) = \mathrm{q}^{-1}$$

Manual control

To get some feeling for how the tank process dynamics reacts for different inputs, try to first control the process manually. The control signal used is displayed in the first window as a P-controller with gain K and a bias level u_0. The value u_0 can be used to set the equilibrium point.

- Choose the gain of the P-controller $K = 0$ to turn off the proportional feedback. The process in now running in open loop. It will react upon your manual input from u_0. Try to fill the tank by increasing u_0 and avoid getting overflow! The input is limited to a maximum flow and also the minimum flow is zero (the pump cannot suck up water).

- Try to find the equilibrium u_0 corresponding to the reference level mark as a red arrow at the tank. Then after the equilibrium is reached, click on the red valve to open it. More water is then flowing out of the tank and you must then compensate this disturbance by increasing u_0.

Problem 1 — P-control

a) Close the valve and choose proportional feedback by increasing K. The control signal is now $u = u_0 + K(r - y)$. The difference equation description considers deviation from the equilibrium corresponding to u_0, thus where the input (deviation from u_0) is $u = K(r - y)$. The closed-loop characteristic polynomial is then $A_c = A + BK$ and the closed loop is

$$y(k) = \frac{BK}{A + BK} r(k) = \frac{K\mathrm{q}^{-1}}{1 - (1 - K)\mathrm{q}^{-1}} r(k)$$

Choose *dead-beat* control $K = 1$ to make the output track the reference with just one sample delay $y(k) = r(k - 1)$. Verify this behavior by making a step change in the reference level r (click and move the red arrow). The step must of course be small enough not to cause the control signal to saturate ($0 \leq u \leq 100$) .

b) Choose different gains to verify that the closed-loop system behaves like a first order system. The response should be monotonous for $K < 1$ and starting to oscillate for $K > 1$ (why?).

c) Adjust u_0 to make $y = r$. Then open the valve by clicking on the red circle. This can be considered as a step disturbance entering somehow at the process. Thus, the model is

$$\begin{aligned} Ay &= Bu + Cd \\ u &= K(r - y) \end{aligned} \qquad \rightarrow y = \frac{BK}{A_c} r + \frac{C}{A_c} d$$

As a result of the disturbance step (opening of the valve) there will be a bias (stationary error). Verify this!

d) Perform a Ziegler-Nichols experiment: Tune the gain K of the P-controller such that the system is on the stability boundary, i.e. oscillates with constant amplitude. This is the critical gain K_c and the corresponding critical period of the oscillation is T_c (number of samples). From this information the tuning of a PI controller should be (according to the Z-N tuning rule):

$$K = K_c \cdot 0.45$$
$$T_i = T_c/1.2$$

Problem 2 —- PI control

a) Click in the first window in the choice for *PI*. Also, unmark *antiwindup*. The controller is then changed to

$$\mathsf{R}(\mathrm{q}^{-1})u(k) = -\mathsf{S}(\mathrm{q}^{-1})y(k) + \mathsf{T}(\mathrm{q}^{-1})r(k), \qquad \begin{cases} \mathsf{R}(\mathrm{q}^{-1}) = 1 - \mathrm{q}^{-1} \\ \mathsf{S}(\mathrm{q}^{-1}) = s_0 + s_1\mathrm{q}^{-1} \\ \mathsf{T}(\mathrm{q}^{-1}) = \mathsf{S}(\mathrm{q}^{-1}) \end{cases}$$

or equivalently in recursive form

$$u(k) = u(k-1) - s_0 e(k) - s_1 e(k-1), \qquad e(k) = r(k) - y(k)$$

The controller pole is at 1 (integral action) and it also has a zero at $z = -s_1/s_0$. The discretization of the PI-control structure is by backward-difference approximation, which gives

$$s_0 = K(1 + 1/T_i)$$
$$s_1 = -K$$

The parameterization of the controller makes it possible to change (interactively) the gain K and the integration time T_i. Notice that u_0 is not needed because of the integral action. The closed-loop characteristic polynomial is

$$\mathsf{A}_c = \mathsf{AR}+\mathsf{BS} = (1-\mathrm{q}^{-1})^2+\mathrm{q}^{-1}(s_0+s_1\mathrm{q}^{-1}) = 1-(2-s_0)\mathrm{q}^{-1}+(1+s_1)\mathrm{q}^{-2}$$

Dead-beat design corresponds to placing both poles at the origin, i.e. $\mathsf{A}_c = (1-\lambda_1\mathrm{q}^{-1})(1-\lambda_2\mathrm{q}^{-1}) = 1$. This is achieved here by choosing

$$\begin{cases} K = 1 \\ T_i = 1 \end{cases}$$

The closed-loop response is for this tuning

$$y(k) = \frac{\mathsf{BT}}{\mathsf{A}_c}r(k) + \frac{\mathsf{CR}}{\mathsf{A}_c}d(k) = r(k-1) + \mathsf{C}[d(k) - d(k-1)]$$

Thus, a step disturbance d will be eliminated in only one sample (if u does not saturate!). Open the valve to verify this behavior. Also, the reference is tracked in one sample if the step change is small enough not to cause the control signal to saturate. Verify this!

b) Tune the controller according to the Ziegler-Nichols setting in **Problem 1d)**. Calculate the closed-loop poles λ_1 and λ_2 and A_c. Then calculate the closed-loop response (using the built-in function `filter`). Compare your calculated response to the real animated response.

c) Make a large setpoint change, for example try to fill up to a level very close to the top. Also, try to empty from a full tank to a level close to the bottom. The limitation of the control signal causes integrator windup. What is the resulting response?

d) Click in the first window to choose *anti-windup* and repeat the experiment above. What is the difference in performance?

e) Click in the first window to choose *PI (T scalar)*. The controller T-polynomial is then changed to the scalar $T = S(1) = s_0 + s_1$. Try different controller (Z-N tuned and dead-beat) and verity that the overshoot in the step response disappears. Explain why!

Report

Document what you do and try to explain the behavior of the controlled tank process. Remember that the dynamics that are simulated models the nonlinear nature of the tank process. Thus, the simplified linear model that is described and used in the analysis above is only an approximation. You may find the responses to look different when controlling with little water in the tank than when having a lot of water.