Tank modeling — sampling and identification

Tank dynamics

Download the sysquake file `Tank.sq` and open it in sysquake. A tank process is there animated and can be controlled manually or by feedback control. The dynamics of the process can be described by the mass balance equation

$$A\frac{dh}{dt} = q_{in} - q_{out}$$

where A is the tank area, h is the height of water, q_{in} and q_{out} are the inflow and outflow of water, respectively. Energy balance (Bernoulli) gives

$$\rho g h = \frac{\rho v^2}{2} \quad \Rightarrow v = \sqrt{2gh}$$

where v is the outlet water velocity. With outlet area a, the outflow then becomes

$$q_{out} = av = a\sqrt{2gh}$$

The pump flow is proportional to the pump voltage V, according to

$$q_{in} = kV$$

The nonlinear tank dynamics are therefore

$$A\frac{dh}{dt} = kV - a\sqrt{2gh} \qquad \rightarrow \frac{dh}{dt} = -\alpha\sqrt{h} + \beta V = f(h, V)$$

where $\alpha = \frac{a\sqrt{2g}}{A}$ and $\beta = \frac{k}{A}$. For simplicity, the values are chosen $\alpha = \beta = 1$ and when the valve is opened $\alpha = 4$.

Linearization

Equilibrium ($\dot{h} = 0$) for a constant pump voltage V_0 corresponds to the level $h_0 = V_0^2$. Linearization around the equilibrium gives

$$\frac{d\Delta h}{dt} = f(h, V) \approx \frac{df}{dh}(h_0, V_0)\Delta h + \frac{df}{dV}(h_0, V_0)\Delta V$$

where with notation $y = \Delta h = h - h_0$ and $u = \Delta V = V - V_0$

$$\frac{dy}{dt} = py + du, \qquad \begin{cases} p = -\frac{\alpha}{2\sqrt{h_0}} \\ d = 1 \end{cases}$$

Sampling

Zero-order-hold sampling with sampling period $h_s = 1$ gives

$$y(k) = \lambda y(k-1) + bu(k-1), \qquad \begin{cases} \lambda = e^{ph_s} = e^p \\ b = (\lambda - 1)/p \end{cases}$$

In polynomial form $A(\mathrm{q}^{-1}) = 1 - \lambda \mathrm{q}^{-1}$ and $B(\mathrm{q}^{-1}) = b\mathrm{q}^{-1}$.

Identification

The parameters λ and b can be estimated experimentally by the least-squares method. This is performed as follows: Excite the system around the equilibrium h_0, collect input and output data, solve the least-squares problem as described below. Collect 100 data samples and form the equation system (after eliminating the mean of all signals)

$$\begin{pmatrix} y(2) \\ \vdots \\ y(100) \end{pmatrix} = \begin{pmatrix} y(1) & u(1) \\ \vdots & \vdots \\ y(99) & u(99) \end{pmatrix} \begin{pmatrix} \lambda \\ b \end{pmatrix} + \begin{pmatrix} e(2) \\ \vdots \\ e(100) \end{pmatrix}$$

or in matrix form $\mathbf{y} = W\theta + \mathbf{e}$, where $\mathbf{e}$ is a vector of equation errors. Minimization of the squared equation error $\sum e(k)^2 = \mathbf{e^T e}$ results in the analytical solution

$$\theta = (W^T W)^{-1} W^T y$$

Problem 1 — Tank models at different operating points

Now do the following experiment with the tank process. To access the samples, first write in the command window

```
> global samples
```

The variable `samples` will now continuously change with the data from the simulation. It contains 100 of the latest data samples, as `samples=[t, u, y]`, where t is the time (in seconds), u the input samples and y the output samples (height level h).

a) Excite the tank system manually using u_0 ($K = 0$) such that y varies with mean close to $h_0 = 20$. After 100 seconds (samples) store data into a variable and form the equation system:

```
> s=samples;
> y=s(2:100,3); y1=s(1:99,3); u=s(2:100,2); u1=s(1:99,2);
> y=y-mean(y); y1=y1-mean(y1); u=u-mean(u); u1=u1-mean(u1);
> W = [y1 u1]; th = (W'*W)\W'*y
```

Calculate $\theta = [\lambda, \quad b]^T$ from sampling of the system. Is the estimated θ close to this theoretical one?

b) Calculate the response of your estimated model and compare it to the real response. Do as follows:

```
> B_1a=[0 th(2)]; A_1a=[1 -th(1)];
> ye=filter(B_1a,A_1a,u); plot([ye y]','rb');
```

The blue curve is the real response and the red one the estimated model response.

c) Repeat the above experiment with excitation around $h_0 = 350$. Calculate the theoretical θ and compare it to what you can estimate experimentally.

Problem 2 — Changed model when opening the valve

Click on the valve to opening it. Then the outflow is increased and $\alpha = 4$. Estimate how the model is changed by repeating Problem **1a** and **b**, now with the valve fully opened.

Report

Answers to the problems above should be clearly documented in a report with comments and evaluations. Include figures of your experiments.