Practical design criteria

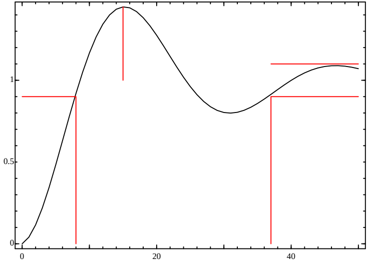
Outline

1. Classical criteria
   - Time response constraints
   - Frequency response constraints

2. Sensitivity criteria
   - Sensitivity functions
   - Shaping of sensitivity functions
   - Multi-model pole placement

Outline

Definitions



Characterizations of a step response (example in parenthesis)

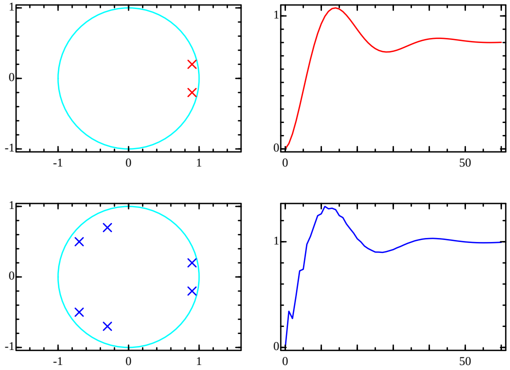
Overshoot: Maximum minus setpoint in percent (45%)

Settling time: to 10% maximum deviation from setpoint (37)

Rise time: Minimum time to reach 90% of setpoint (8)

Dominating poles

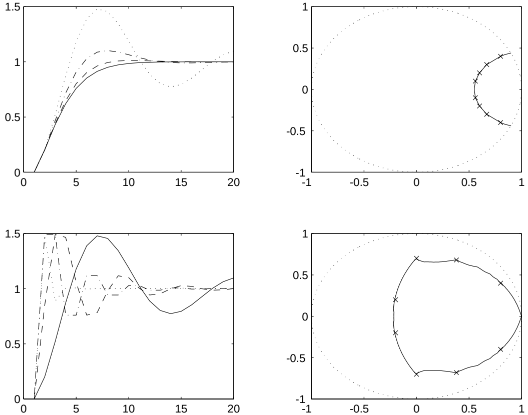
Poles closest to 1 determines characteristic shape of step response



Pole position versus overshoot and speed

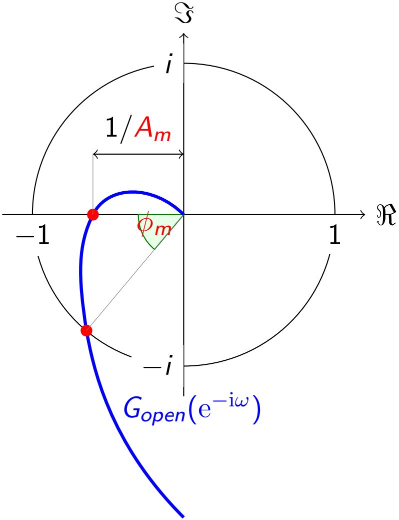
Study step response of system

$$G(\mathrm{q}^{-1}) = \frac{B}{A} = \frac{A(1)\mathrm{q}^{-1}}{(1-\lambda \mathrm{q}^{-1})(1-\lambda^{*}\mathrm{q}^{-1})}$$



Outline

Amplitude and phase margins



Nyquist curve in complex plane

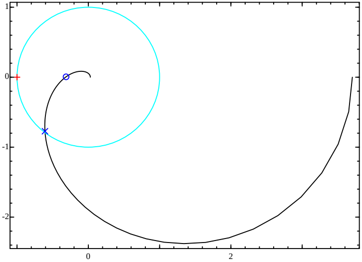
$$G_{open}(\mathrm{e}^{-\mathrm{i}\omega}), \quad \omega = 0 \to \pi$$

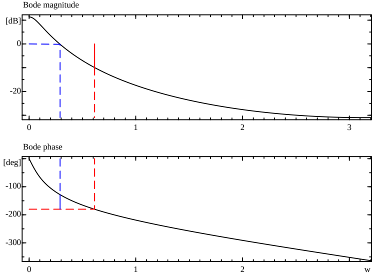
Closed loop stability margins

Amplitude (gain) margin: A_m

Phase margin: ϕ_m

Margins in Nyquist and Bode diagram





Classical design in frequency domain by sequence of lead-lag filters

$$C = C_1 C_2 \dots$$

shapes the (open-loop) frequency function to obtain good margins

Classical lead-lag design

Benefits

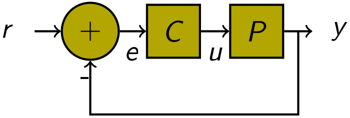
+ The robustness margins make sense
+ Model-based design (constructive, not ad hoc)

Drawbacks

- Awkward with two margins in two curves
- Unnecessary high order controller
- No control over dominant poles (time response)

Outline

Closed-loop sensitivity for process variations



$$y = Hr, \quad H = \frac{PC}{1+PC}$$

$$\frac{dH}{dP} = \frac{C(1+PC) - CPC}{(1+PC)^2} = \frac{H}{P}\frac{1}{1+PC}$$

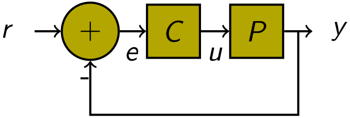
Relative sensitivity

$$\frac{dH}{H} = \underbrace{\frac{1}{1+PC}}_{S_y}\frac{dP}{P}$$

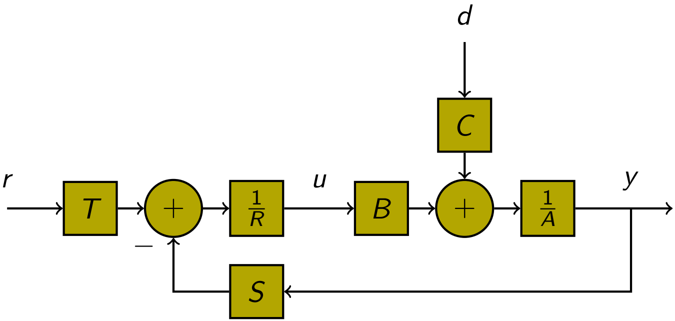
(Output) Sensitivity function

$$S_y = \frac{1}{1+PC}$$

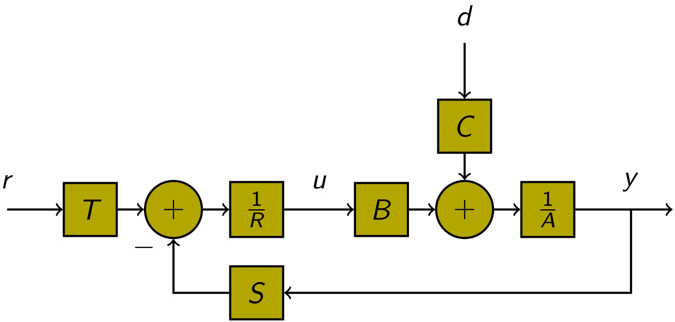
Loop transfer



Loop transfer $L = PC = \frac{BS}{AR}$



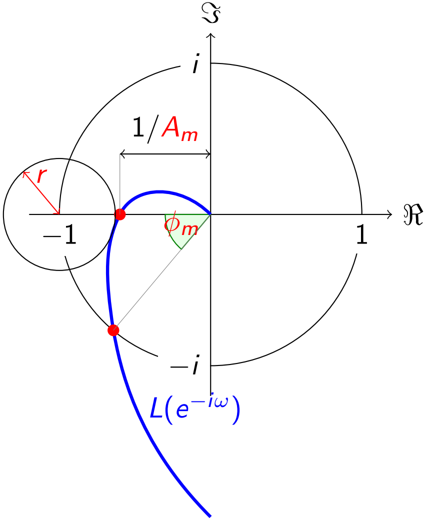
Output sensitivity function



Output disturbance $C = A$ related to output sensitivity

$$y = \frac{AR}{A_c} d = S_y d$$

Relation between Nyquist curve and output sensitivity

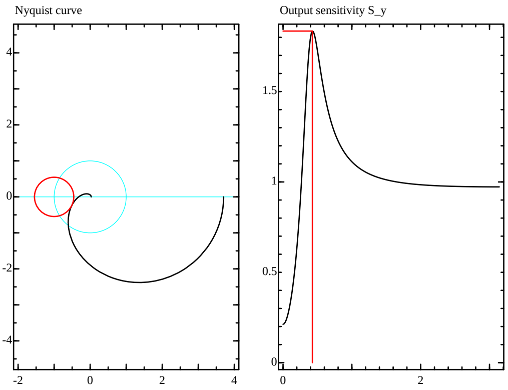


$$\max_\omega |S_y(e^{-i\omega})| = \max \frac{1}{|1+L(e^{-i\omega})|} = \frac{1}{\min |1+L(e^{-i\omega})|} = \frac{1}{r}$$

One margin ensuring two

$$\begin{cases} A_m \geq \frac{1}{1-r} \\ \phi_m \geq 2\arcsin(\frac{r}{2}) \end{cases}$$

Example



Nyquist curve avoid circle with radius
$r = 1/\ max|S_y(e^{-i\omega})| = 1/1.83 = 0.55$

Input sensitivity function

Input response to output disturbance ($C = A$)

$$u = -\frac{SA}{A_c}d$$

Define Input sensitivity function

$$S_u = -\frac{SA}{A_c}$$

Should be bounded at high frequencies to reduce noise feedback

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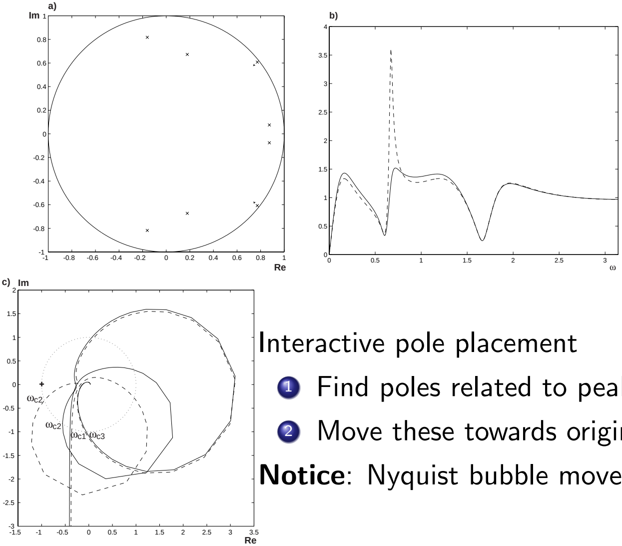
Shaping of sensitivity functions



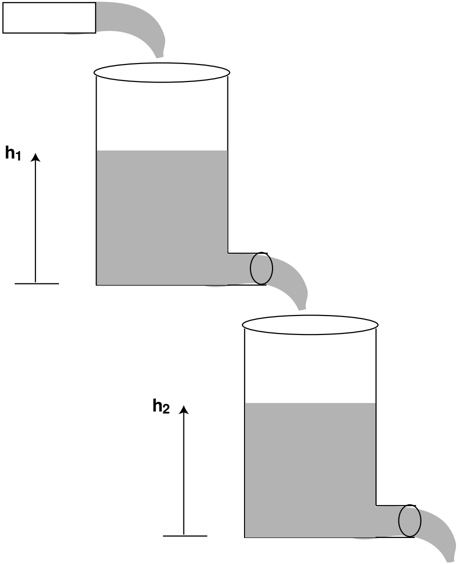
Interactive pole placement

1. Find poles related to peak in S_y ($\arg \lambda = \omega$)
2. Move these towards origin

Notice: Nyquist bubble moves away from -1

Outline

Example



Mass balance

$$A\frac{dh}{dt} = q_{in} - q_{out}$$

Energy balance

$$\rho gh = \frac{\rho v^2}{2} \quad \Rightarrow v = \sqrt{2gh}$$

Outflow

$$q_{out} = av = a\sqrt{2gh}$$

Pump flow

$$q_{in} = kV$$

Example: dynamical model

Nonlinear tank dynamics

$$\begin{array}{l} A\frac{dh_1}{dt} = kV - a\sqrt{2gh_1} \\ A\frac{dh_2}{dt} = a\sqrt{2gh_1} - a\sqrt{2gh_2} \end{array} \rightarrow \left\{ \begin{array}{l} \frac{dh_1}{dt} = -\alpha\sqrt{h_1} + \beta V \\ \frac{dh_2}{dt} = \alpha\sqrt{h_1} - \alpha\sqrt{h_2} \end{array} \right.$$

Assume $\alpha = \frac{a\sqrt{2g}}{A} = \beta = \frac{k}{A} = 1$
Linearize around equilibrium h^0

$$x = \begin{pmatrix} h_1 - h^0 \\ h_2 - h^0 \end{pmatrix}, \quad u = V - \sqrt{h^0}, \quad y = h_2 - h^0$$

$$\left\{ \begin{array}{l} \dot{x} = Ax + Bu = \begin{pmatrix} -c & 0 \\ c & -c \end{pmatrix} x + \begin{pmatrix} 1 \\ 0 \end{pmatrix} u \\ y = Cx = \begin{pmatrix} 0 & 1 \end{pmatrix} x \end{array} \right.$$

$G(s) = C(sI - A)^{-1}B = \frac{c}{(s+c)^2}$, where $c = \frac{1}{2\sqrt{h^0}}$

Example: Study two different linearization points

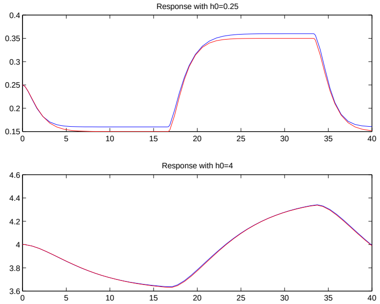
With $h^0 = 1/4$

$$G_1(1) = \frac{4}{(s+4)^2}$$

With $h^0 = 4$

$$G_2(s) = \frac{1/4}{(s+1/4)^2}$$

Nonlinear and linear responses



Multi-model pole placement

Open polp.sq in Sysquake and write

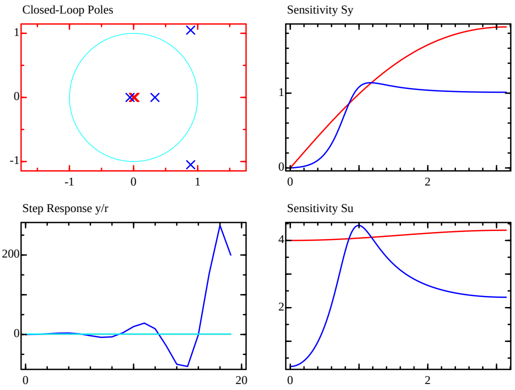
$$(B1, A1) = c2dm(4, poly([-4, -4]), 1)$$
$$(B2, A2) = c2dm(1/4, poly([-1/4, -1/4]), 1)$$

Change to backward-shift representation

$$B1 = [0, B1]$$
$$B2 = [0, B2]$$

In **Settings-System**, change $[B1; B2], [A1; A2]$ (write digits manually)
In **Settings-Fix factor** R_f, change to $[1, -1]$ (integral action)

Sysquake: start of design



Sysquake: after some moves of poles