Servo tracking control

Simplified servo dynamics with dry friction

Servo dynamics with dry friction is describe as

$$\dot{x}(t) = f(u(t))$$

where x is the servo position, u control input (voltage) and f defined as

$$f(u) = \mathrm{sign}(u) \cdot \max(|u| - d, 0);$$

When the input is increasing from zero the servo output is not moving until the input exceeds the friction force parameter d (here $d = 0.1$). Thus, the servo is stuck and cannot move until enough input is generated.

Discrete-time model

Without the friction ($d = 0$), zero-order-hold sampling gives the linear model

$$x(k) = \frac{h\mathrm{q}^{-1}}{1 - \mathrm{q}^{-1}} u(k)$$

For inputs that exceeds the friction the system behaves as if there were a constant disturbance d. Thus, it should be possible to eliminate it with integral action, at least as long as u does not changes sign. If that happens. the disturbance also changes sign. The linear model can therefore be described as (using $h = 1$)

$$(1 - \mathrm{q}^{-1})x = \mathrm{q}^{-1}(u + d), \quad d = \begin{cases} 0.1 & u > 0.1 \\ -0.1 & u < -0.1 \end{cases}$$

Problem 1 — Tracking constant references

Investigate the following different controller designs for tracking a manually chosen constant set-point. Open the Sysquake-file `Servo.sq`. The servo model described above is there controlled by the controller of the standard structure

$$Ru = -Sy + Tr$$

When opening the file, default is manual control which means that $R = T = 1$ and $S = 0$, i.e. $u = r$. The reference can be manipulated with the mouse by clicking on the black circle and moving it horizontally. The red circle is the servo output. Try first to control the system manually to get the feeling for how it responds. Whenever your input is absolute less than 0.1 the servo stops. Now try automatic control designs. Click on the *Setting* menu and choose *Controller*. The present controller polynomials coefficients are then shown and you can just edit there to change controller. In the following designs you should solve polynomial equations. You may use a function for this. Open `polp.sq` and write in the command window

```
> use _SQ_polp
```

All functions within polp.sq is now available from the command window. One of these, solves the polynomial equation. It is defined as

```
> (X,Y)=peq(A,B,C)
```

and calculate the polynomials X and Y from the polynomial equation $AX + BY = C$, where A, B, C are polynomials in backward-shift representation (i.e. $A = 1 + a\mathrm{q}^{-1}$ is represented as $A = [1 \quad a]$.) Calculate the following controllers expressed as R, S and T and input their coefficients in the Setting menu (Controller) to try them. Choose T as scalar to adjust for steady-state gain one. Comment the resulting performance.

a) P-controller with $A_c = 1$ (dead-beat design).

b) A controller with integral action and $A_c = 1$.

Problem 2 — Tracking a triangular-like reference

Change in the *Setting* menu to *Setpoint Generation*. A triangular-like reference is now generated. It is a Fourier-series approximation of a triangular wave and described as

$$r(k) = \frac{N}{D}\delta(k) \qquad D = D_1 D_2$$

where $D_i = 1 - 2\cos(\omega_i)\mathrm{q}^{-1} + \mathrm{q}^{-2}$, $i = 1, 2$ and $\omega_1 = 2\pi/20$, $\omega_2 = 3\omega_1$. Neither of the above controllers will now be able to track the moving reference without error. This is partly because the T-polynomial needs to be redesigned and partly because of the disturbance.

a) Change the T-polynomial in controller **1b** above, and design it based on the annihilation polynomial D.

b) Use the calculated T above but change the design for R and S for robust tracking. Thus, because of the reference shape the disturbance can be modeled by the annihilation polynomial $R_f = 1 + \mathrm{q}^{-10}$, since $d(k) = -d(k-10)$. Consequently, use R_f as fix factor in R. Choose $A_c = 1$.

Report

Document you findings about performance for the different designs in a report.