Exam for Random Processes, 7.5 ECTS

August 12, 2009, 9.00 – 13.00

Max number of points: 30.
Halmstad University grading bounds: 12p ⇒ grade 3, 18p⇒ grade 4, 24p ⇒ grade 5.
Allowed aids: Summary of formulae attached to the exam, calculator and Math. Handbook: Beta.
Examiner: Eric Järpe (035-16 76 53, 0702-822 844).

For each problem a *complete* solution should be given. All solutions should be thoroughly presented. Each solution should start at the top of a new sheet of paper. Only one solution a sheet. The proper solutions will be available on the internet at
`http://dixon.hh.se/erja` → `Teaching` → `Random processes` → `Previous exams`

1. Show that the spectral density of an AR(p) process is

$$R(f) = \frac{\sigma_\epsilon^2}{|1+\sum_{k=1}^p a_k e^{-i2\pi fk}|^2} \qquad (4p)$$

2. Let $\{N_t : t \in \mathbb{R}^+\}$ be a Poisson process with intensity 3, and let $\{Y_t : t \in \mathbb{R}^+\}$ be defined by $Y_t = N_t + N_{t+2}$. What is the covariance function of Y_t? (3p)

3. Calculate the covariance function of a shot noise process with parameter 0.05 and pulse function $g(t) = I(t \in [0,1])$. (4p)

4. Assume that the weakly stationary process $\{X_t\}$ is observed to be

   10.3 11.2 10.9 9.1 9.3

   at times $t = 1, 2, 3, 4, 5$, and that it is known that $E(X_t) = 10$. Determine an unbiased estimator of the covariance function of $\{X_t\}$. (3p)

5. Assume that $\{X_t : t \in \mathbb{R}\}$ is weakly stationary with covariance function $g(a\tau - b)$ where $a \neq 0$. Determine the spectral density of $\{X_t\}$ in terms of $G = \mathcal{F}(g)$. (4p)

6. Consider the weakly stationary process $\{\theta_t : t \in \mathbb{R}\}$ with spectral density $R(f) = e^{-f^2}$. How closely should observations of $\{\theta_t\}$ be sampled to make the risk for aliasing smaller than 5%? (4p)

7. A strongly stationary Gaussian process $\{X_t : t \in \mathbb{R}\}$ has covariance function $r(\tau) = e^{-|\tau|}$.
   (a) Calulate $P(X_t - X_{t+0.5} > 1)$. (3p)
   (b) Assume that $\{X_t\}$ is filtered with transfer function

$$H(f) = \begin{cases} (1+(2\pi f)^2)^{-1/2} & |f| < 1 \\ 0 & \text{otherwise} \end{cases}$$

   Show that 4 is an upper bound for the variance of the filtered signal. (5p)

GOOD LUCK!