

# EXAM FOR STOCHASTIC MODELS IN DISCRETE TIME

## 3.75 ECTS

Master's program of Financial Mathematics

October 30, 2009, 9.00 – 13.00

**Max number of points:** 30.

**Halmstad University grading bounds:** 12p  $\Rightarrow$  grade 3, 18p  $\Rightarrow$  grade 4, 24p  $\Rightarrow$  grade 5.

**ECTS bounds:** 12p  $\Rightarrow$  grade E, 15p  $\Rightarrow$  grade D, 18p  $\Rightarrow$  grade C, 21p  $\Rightarrow$  grade B, 24p  $\Rightarrow$  grade A.

**Allowed aids:** Summary of formulae attached to the exam, calculator and dictionary.

**Examiner:** Eric Järpe (035-16 76 53, 0702-822 844).

For each problem a *complete* solution should be given. All solutions should be thoroughly presented.

Each solution should start at the top of a new sheet of paper. Only one solution a sheet.

The proper solutions will be available on the internet at <http://www.hh.se/staff/erja>  $\rightarrow$  Teaching

$\rightarrow$  Financial Mathematics  $\rightarrow$  Stochastic models  $\rightarrow$  Previous exams

1. Let  $X = \{(X_n, \mathcal{F}_n) : n \geq 0\}$  be a stochastic sequence with  $E(|X_0|) < \infty$ . Prove that if  $X$  is a martingale transformation, then  $X$  is a local martingale. (4p)

2. Assume  $\{X_n\}$  is an  $MA(2)$  process with parameters  $b_0 = b_1 = b_2 = \sigma_\epsilon^2 = 1$ . Calculate

(a) the variance  $D(X_n)$  (3p)

(b) the probability  $P(X_n > X_{n+1} + 1)$  (4p)

3. Let  $\{\xi_k\}_{k=0}^\infty$  be a classical random walk starting in 0 at time 0.

(a) Prove that  $\{\xi_k\}$  is non-stationary. (4p)

(b) Calculate the probability that the random walk returns to zero at time  $n$ . (5p)

4. (*Cusum procedure*) Let  $\{X_k\}$  be a sequence of independent identically distributed random variables with zero mean, and let the sequence  $\{S_n\}$  be defined by

$$S_n = \max_{k=1,2,\dots,n} \sum_{j=k}^n X_j$$

(a) Derive a recursive representation of  $\{S_n\}$ , i.e. find a relationship  $f$  such that, with  $S_1 = X_1$ , we have  $S_n = f(X_n, S_{n-1})$  for  $n = 2, 3, \dots$  (3p)

(b) Prove that  $\{S_n\}$  is a submartingale with respect to the flow  $\mathcal{F}_n = \sigma(X_1, X_2, \dots, X_n)$ . (2p)

5. Assume that  $\{h_t\}$  is a stochastic volatility process of order 1 with parameters  $a_0, a_1$  such that  $0 < a_1 < 1$  and  $c = 1$ . Prove that the fourth moment of  $h_t$  is bounded from above by  $3e^2$ . (5p)

*GOOD LUCK!*