Lösningsförslag Ellära 2012-05-25

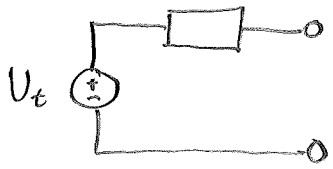
① $R_i = \frac{U_t}{I_k}$

$I_k = \{$ ur graf $\} = 4{,}5$ A kortslutningsströmmen

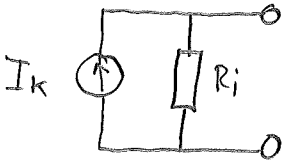
$U_t = \{$ ur graf $\} = 1{,}4$ volt Tomgångsspänningen

$R_i = \frac{U_t}{I_k} = \frac{1{,}4}{4{,}5} = 0{,}311\ \Omega$

Sp. 2 pol R_i

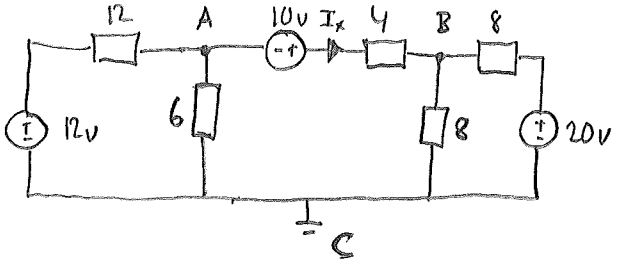


Ström 2 pol



---

② a) I_x med nodanalys



Kl nod A (alla strömmen ut)

$$\frac{V_A - 12}{12} + \frac{V_A}{10} + \underbrace{\frac{V_A - V_B + 10}{4}}_{I_x} = 0 \quad ①$$

kl nod B (alla strömmen ut)

$$\frac{V_B - V_A - 10}{4} + \frac{V_B}{8} + \frac{V_B - 20}{8} = 0 \quad ②$$

① $V_A\left(\frac{1}{12} + \frac{1}{6} + \frac{1}{4}\right) - \frac{12}{12} + \frac{10}{4} - \frac{V_B}{4} = 0$

$0{,}5 \cdot V_A + 1{,}5 = 0{,}25\, V_B$

② $V_B\left(\frac{1}{4} + \frac{1}{8} + \frac{1}{8}\right) - \frac{10}{4} - \frac{20}{8} - \frac{V_A}{4} = 0$

$V_B \cdot 0{,}5 - 5 - 0{,}25 V_A = 0$

$\Rightarrow V_B = 0{,}5\, V_A + 10$

① + ②

$0{,}5\, V_A + 1{,}5 = 0{,}25 \cdot (0{,}5\, V_A + 10)$

$0{,}5\, V_A - 0{,}125\, V_A = 2{,}5 - 1{,}5 = 1$

$V_A = 2{,}67$ Volt $\frac{8}{3}$

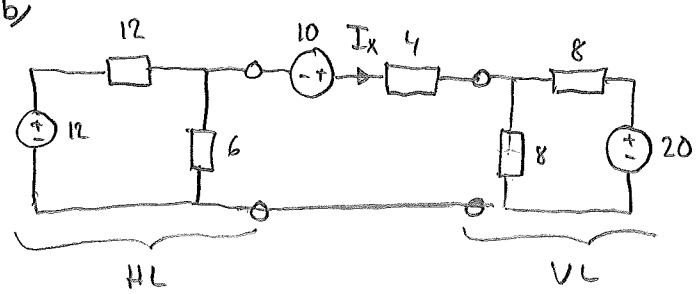
$V_B = 11{,}3$ volt $\frac{68}{6}$

$$I_x = \frac{V_A - V_B + 10}{4} = \frac{2{,}67 - 11{,}3 + 10}{4} = \underline{0{,}34\ A}$$

$$I_x = \frac{\frac{8 \cdot 2}{3 \cdot 2} - \frac{68}{6} + \frac{60}{6}}{4} = \frac{\frac{8}{6}}{4} = \frac{8}{24} = \underline{0{,}33\ A}$$

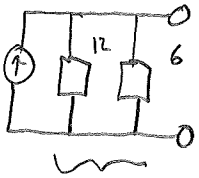
Lösningsförslag Ellära 2012-05-25

(2) b/



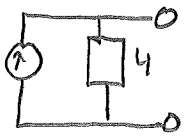
(HL)

$I = \frac{U}{R} = \frac{12}{12} = 1$

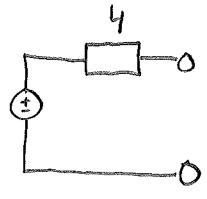


$12 // 6 = 4$

$\Rightarrow$

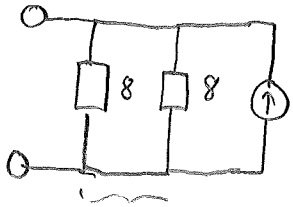


$\Rightarrow$

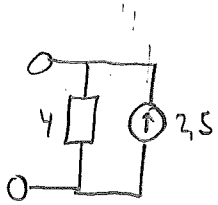


$U = I \cdot R$
$= 1 \cdot 4$
$= 4$

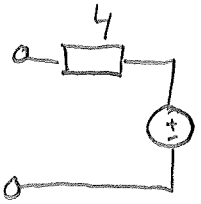
(VL)



$I = \frac{U}{R} = \frac{20}{8} = 2{,}5$ $\Rightarrow$



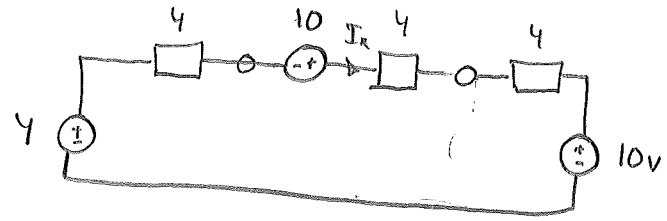
$\Rightarrow$



$U = R \cdot I$
$= 4 \cdot 2{,}5 = 10V$

$8 // 8 = 4\ \Omega$

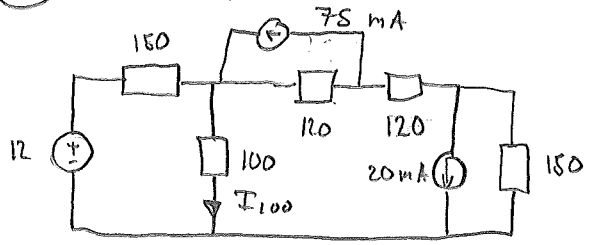
Ekv. schema



$I_x = \frac{U}{R} = \frac{10 + 4 - 10}{4+4+4} = \frac{1}{3} = \underline{\underline{0{,}33\ A}}$

Lösningsförslag Ellära 2012-05-25

(3) Effekten $P = U \cdot I = R \cdot I^2$ $\Rightarrow$ Bestäm I_{100} (Alt Bestäm U_{100})

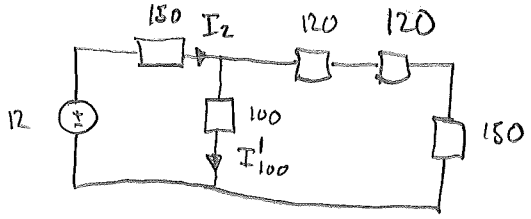


Nollställning av generatorer

sp. generator → kortslutning

Ström generator → avbrott

1) Nollställ alla generatorer utom 12 volt

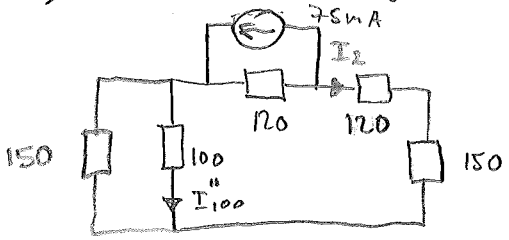


$R_{tot} = 150 + 100 // \underbrace{(120 + 120 + 150)}_{390} = 229{,}6$

$I_2 = \frac{12}{R_{tot}} = 52{,}26 \text{ mA}$

$I'_{100} = \{\text{Strömdelning}\} = \frac{390}{100 + 390} \cdot I_{tot} = \underline{41{,}6 \text{ mA}}$

B) Nollställ alla generatorer utom 75 mA

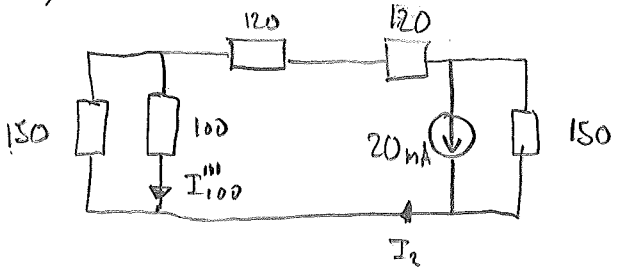


Strömdelning ger:

$I_2 = \frac{120}{120 + 120 + 150 + \underbrace{100 // 150}_{60}} \cdot 75 = 20 \text{ mA}$

$I''_{100} = \frac{150}{100 + 150} \cdot I_2 = \underline{12} \text{ mA}$

C) Nollställ alla generatorer utom 20 mA



Strömdelning ger.

$I_2 = \frac{150}{150 + 120 + 120 + \underbrace{100 // 150}_{60}} \cdot 20 \text{ mA} = 6{,}667 \text{ mA}$

$I_{100} = \frac{150}{100 + 150} \cdot I_2 = \underline{-4} \text{ mA}$

↑ obs tecken

$I_{100} = I'_{100} + I''_{100} + I'''_{100} = 49{,}6 \text{ mA}$

$P = R \cdot I_{100}^2 = 100 \cdot (49{,}6 \cdot 10^{-3})^2 = \underline{2{,}46 \text{ mW}}$

④ a) $z = \frac{U}{I}$; $U = |U| e^{j(\omega t + \alpha)}$

$I = |I| e^{j(\omega t + \beta)}$

Från figuren fås:

× $|U| = 5V$; $\alpha = 0$; $T = 0{,}1\,ms \Rightarrow f = 10\,kHz$ och $\omega = 2\pi f$

× $|I| = 30mA$; $\beta = -\frac{1}{5} \cdot 360° = -72°$

$$\Rightarrow z = \frac{5}{30 \cdot 10^{-3}} e^{j(\alpha - \beta)} \approx 167\, e^{j72°}$$

$$= 167\,(\cos(72°) + j\sin(72°)) = (51{,}6 + j\,158{,}8)\,\Omega$$

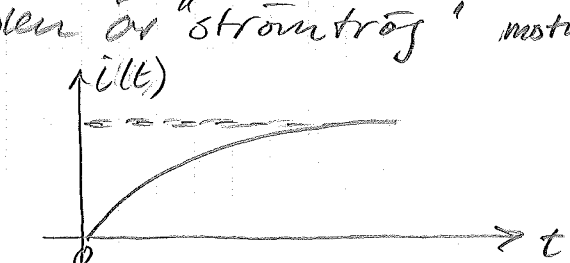
b) $z = R + jX$; $X = \omega \cdot L$

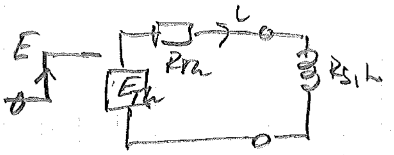
$\Rightarrow R = 51{,}6\,\Omega$

$\omega L = 158{,}8 \Rightarrow L = \frac{158{,}8}{2\pi \cdot 10^4} \approx 2{,}5\,mH$

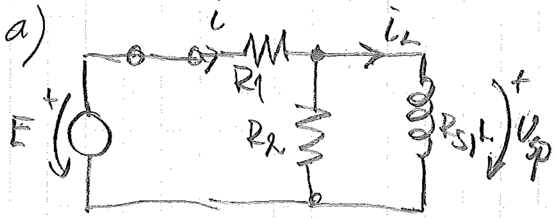
---

⑤ Vid tiden t=0 sluts switchen (E + switch ⇔ E steg) och spolen ($R_s + j\omega L$) laddas upp.
Spolen är "strömtrög" motverkar strömändringar!





a)

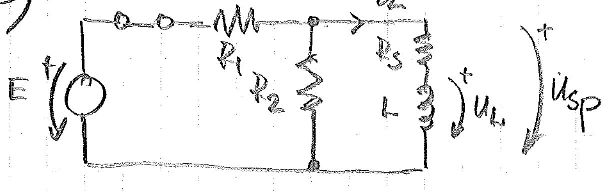


$i_L(0) = 0$ se i(t) ovan!

$$\Rightarrow U_{sp}(0) = E \cdot \frac{R_2}{R_1 + R_2} = 60 \cdot \frac{60}{90} = 40V$$

b) $$i(0) = \frac{E}{R_1 + R_2} = \frac{60}{90} \approx 0{,}67\,A$$

c)

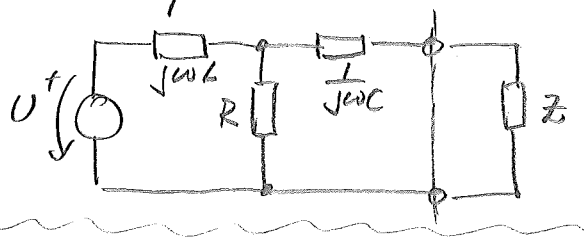


Efter lång tid är i_L konstant (se i(t) ovan!) och $u_L = L \cdot \frac{di}{dt} = 0$

$$\Rightarrow U_{sp}(\infty) = E \cdot \frac{R_s // R_2}{R_s // R_2 + R_1}$$

$$= 60 \cdot \frac{20//60}{20//60 + 30} = 60\,\frac{15}{45} = 20V$$

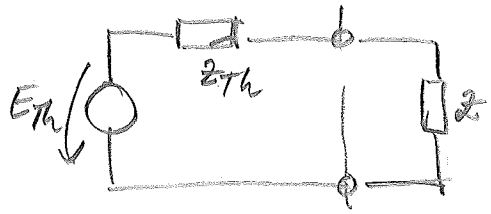
(6) Komplext schema:



$U = 90{,}0\, e^{j\omega t}$ V ; $\omega = 10$ krad/s
$R = 60\,k\Omega$
$L = 6{,}0$ H
$C = 2{,}5$ nF

---

a) Gör om till spännings-tvåpol



$Z = Z_{Th}^*$ för maximal effekt utv.

$$Z_{Th} = R \,//\, j\omega L + \frac{1}{j\omega C} = \frac{R \cdot j\omega L}{R + j\omega L} - \frac{j}{\omega C}$$

$$= \frac{j\,60\cdot10^3 \cdot 10^4 \cdot 6}{60\cdot10^3 + j\,10^4\cdot 6} - \frac{j}{10^4 \cdot 2{,}5\cdot10^{-9}}$$

$$= \frac{6\cdot10^4 \cdot j\,60\cdot10^3}{6\cdot10^4(1+j)} - j\,4\cdot10^4 = \frac{60\cdot10^3\, e^{j90^\circ}}{\sqrt{2}\, e^{j45^\circ}} - j\,4\cdot10^4$$

$$= \frac{60\cdot10^3}{\sqrt{2}} e^{j(45^\circ)} - j\,4\cdot10^4 = (30 - j\,10)\ k\Omega$$

$(30 + j30)\,10^3$

$$\Rightarrow Z = Z_{Th}^* = (30 + j10)\ k\Omega$$

---

b) $P = Re(Z)\cdot I_C^2$; $I_C = \dfrac{E_{Th}^{eff.}}{(Z + Z_{Th})} = \dfrac{E_{Th}^{eff.}}{2\cdot 30k}$

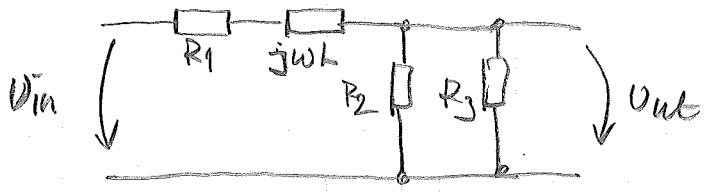
$$E_{Th} = U \cdot \frac{R}{R + j\omega L} = 90\cdot e^{j\omega t}\, \frac{60\cdot10^3}{60\cdot10^3 + j10^4\cdot 6}$$

$$= 90\cdot e^{j\omega t}\, \frac{1}{(1+j)} = \frac{90}{\sqrt{2}}\, e^{j(\omega t - 45^\circ)}$$

$$E_{Th}^{eff} = |E_{Th}|/\sqrt{2} = \frac{90}{\sqrt{2}}/\sqrt{2} = \frac{90}{2} = 45\ V$$

$$\Rightarrow P = 30\cdot10^3 \cdot \left(\frac{45}{2\cdot30\cdot10^3}\right)^2 \approx 16{,}9\ mW$$

(7) Komplekt schema:



$R_1 = R_2 = 1\,k\Omega$
$R_3 = 100\,k\Omega$
$L = 100\,mH$

a) $F(\omega) = \frac{V_{ut}}{V_{in}}$

$V_{ut} = V_{in} \frac{R_2 // R_3}{R_2 // R_3 + R_1 + j\omega L}$; $R_2 // R_3 = 1k // 100k \approx 1k$

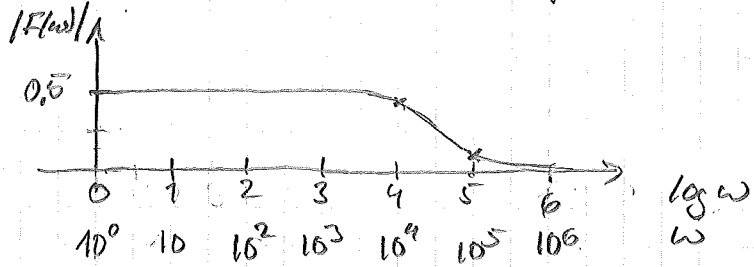
$\Rightarrow F(\omega) = \frac{1k}{1k + 1k + j\omega 0{,}1} = \frac{1k}{2k + j\omega 0{,}1}$

$= \frac{1}{2 + j\omega 10^{-4}}$

b) $\omega = 0 \Rightarrow F(0) = \frac{1}{2}$ och $|F(0)| = 0{,}5$

$\omega = 10^4 \Rightarrow F(10^4) = \frac{1}{2+j}$ och $|F(10^4)| = \frac{1}{\sqrt{5}} \approx 0{,}45$

$\omega = 10^5 \Rightarrow F(10^5) = \frac{1}{2+j10}$ och $|F(10^5)| = \frac{1}{\sqrt{104}} \approx 0{,}1$



c) $|F(\omega_g)| = \frac{0{,}5}{\sqrt{2}}$

$\frac{1}{\sqrt{4 + (\omega \cdot 10^{-4})^2}} = \frac{1}{2\cdot\sqrt{2}}$

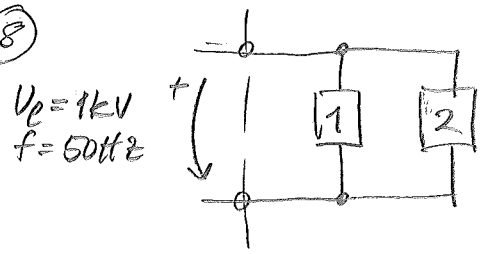
$\Rightarrow 4 + (\omega \cdot 10^{-4})^2 = 8$

$\Rightarrow (\omega \cdot 10^{-4})^2 = 4 \Rightarrow \omega \cdot 10^{-4} = 2$

$\omega = \frac{2}{10^{-4}} = 2 \cdot 10^4$ rad/s

$= 20$ krad/s

(8)

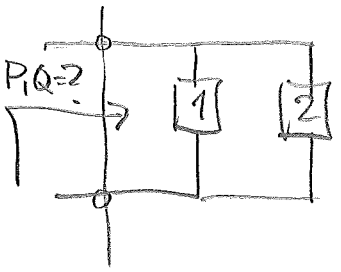


1 : $C = 10\mu F$

2 : $|S| = 10\,kVA$; $\cos(\varphi) = 0{,}80$ (ind).

---

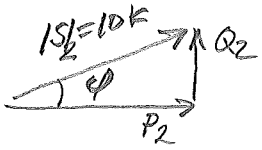
a)



Last 1: (rein kapazitiv!) $P_1 = 0$; $Q_1 = -\omega C \cdot U_e^2 = -2\pi \cdot 50 \cdot 10 \cdot 10^{-6} \cdot (10^3)^2$

$= -3{,}14\ kVAr$.

Last 2: (induktiv)



$P_2 = |S_2| \cdot \underbrace{\cos(\varphi)}_{=0{,}8} = 10k \cdot 0{,}8 = 8\ kW$

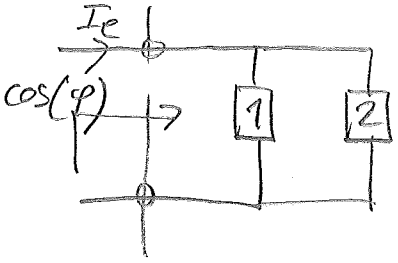
$Q_2 = |S_2| \underbrace{\sin(\varphi)}_{=0{,}6} = 10k \cdot 0{,}6 = 6\ kVAr$

Anlagsgrenzen: $P = P_1 + P_2 = 0 + 8k = \underline{8\ kW}$

$Q = Q_1 + Q_2 = -3{,}14k + 6k = \underline{2{,}86\ kVAr}$

---

b)



$$S = P + jQ = |S| e^{j\varphi} = \underbrace{U_e \cdot I_e}_{=|S|} e^{j\varphi}$$

$$\Rightarrow I_e = \frac{|S|}{U_e} = \frac{|8k + j\,2{,}86k|}{10^3} = \frac{\sqrt{8^2 + (2{,}86)^2} \cdot 10^3}{10^3}$$

$$\approx \underline{8{,}5\,A}$$

$$\cos(\varphi) = \cos\left(\arctan\left(\frac{Q}{P}\right)\right) = \cos\left(\arctan\left(\frac{2{,}86k}{8k}\right)\right)$$

$$\approx \underline{0{,}942}$$