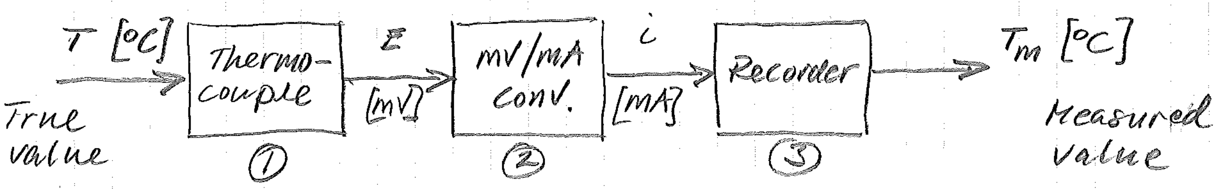
3.1



$T = 117\ °C$
$\sigma_T = 0$

① Thermocouple

Model eq: $\boxed{E = c_0 + c_1 T + c_2 T^2}$; $E = f(c_0, c_1, c_2, T)$

$$\begin{cases} \bar{c}_0 = 0.00 \text{ mV} & ; \ \sigma_{c_0} = 6{,}93 \cdot 10^{-2} \\ \bar{c}_1 = 4{,}017 \cdot 10^{-2} \text{ mV/°C} & ; \ \sigma_{c_1} = 0 \\ \bar{c}_2 = 4{,}66 \cdot 10^{-6} \text{ mV/(°C)}^2 & ; \ \sigma_{c_2} = 0 \end{cases}$$

$$\bar{E} = \bar{c}_0 + \bar{c}_1 \cdot T + \bar{c}_2 \cdot T = 4{,}017 \cdot 10^{-2} \cdot 117 + 4{,}66 \cdot 10^{-6} \cdot (117)^2 = \underline{4{,}7637 \text{ mV}}$$

$$\sigma_E^2 = \left(\frac{\partial E}{\partial c_0}\right)^2 \sigma_{c_0}^2 + \underbrace{\left(\frac{\partial E}{\partial c_1}\right)^2 \sigma_{c_1}^2}_{=0} + \underbrace{\left(\frac{\partial E}{\partial c_2}\right)^2 \sigma_{c_2}^2}_{=0} + \underbrace{\left(\frac{\partial E}{\partial T}\right)^2 \sigma_T^2}_{=0}$$

$$= 1 \cdot (6{,}93 \cdot 10^{-2})^2 = \underline{4.80 \cdot 10^{-3} \ (\text{mV})^2}$$

E is normal distributed $N(\bar{E}, \sigma_E)$.

② mV/mA converter

Model eq: $\boxed{i = K_1 \cdot E + K_M \cdot E \cdot \Delta T_a + K_I \Delta T_a + a_1}$

$$\begin{cases} \bar{K}_1 = 3.893 & ; \ \sigma_{K_1} = 0 \\ \overline{\Delta T}_a = -10 & ; \ \sigma_{\Delta T_a} = 10 \\ \bar{a}_1 = -3{,}864 & ; \ \sigma_{a_1} = 10 \\ \bar{K}_M = 1{,}95 \cdot 10^{-4} & ; \ \sigma_{K_M} = 0 \\ \bar{K}_I = 2{,}00 \cdot 10^{-3} & ; \ \sigma_{K_I} = 0 \end{cases}$$

$\bar{E} = 4.7637$; $\sigma_E^2 = 4{,}80 \cdot 10^{-3}$ from ①

(3.1) cont.

$$\bar{i} = \bar{K}_1 \cdot \bar{E} + \bar{K}_M \bar{E} \cdot \overline{\Delta T_a} + \bar{K}_I \overline{\Delta T_a} + \bar{a}_1$$

$$= 3{,}893 \cdot \bar{E} + 1{,}95 \cdot 10^{-4} \cdot \bar{E}\,(-10) + 2{,}00 \cdot 10^{-3}(-10) - 3{,}864$$

$$\simeq \underline{14{,}652 \text{ mA}}$$

$$\sigma_i^2 = \left(\frac{\partial i}{\partial E}\right)^2 \sigma_E^2 + \left(\frac{\partial i}{\partial \Delta T_a}\right)^2 \sigma_{\Delta T_a}^2 + \left(\frac{\partial i}{\partial a_1}\right)^2 \sigma_{a_1}^2$$

when $\sigma_{K_1} = \sigma_{K_M} = \sigma_{K_I} = 0$

$$= (\bar{K}_1 + \bar{K}_M \overline{\Delta T_a})\,\sigma_E^2 + (\bar{K}_M \cdot \bar{E} + \bar{K}_I)\,\sigma_{\Delta T_a}^2 + 1 \cdot \sigma_{a_1}^2$$

$$\simeq 7{,}27 \cdot 10^{-2} + 8{,}58 \cdot 10^{-4} + 1{,}96 \cdot 10^{-2} = \underline{9{,}32 \cdot 10^{-2}\ (\text{mA})^2}$$

i is normal distributed $N(\bar{i}, \sigma_i)$

---

(3) Recorder

Model eq. $\boxed{T_M = K_2 \cdot i + a_2}$

$$\begin{cases} \bar{K}_2 = 6{,}25 \ ; & \sigma_{K_2} = 0{,}0 \\ \bar{a}_2 = 25{,}0 \ ; & \sigma_{a_2} = 0{,}30 \end{cases}$$

and $\bar{i} = 14{,}652$; $\sigma_i^2 = 9{,}32 \cdot 10^{-2}$ from (2)

$$\bar{T}_M = \bar{K}_2 \cdot \bar{i} + \bar{a}_2 = 6{,}25 \cdot 14{,}652 + 25{,}0 \simeq \underline{116{,}57\ °C}$$

$$\sigma_{T_M}^2 = \left(\frac{\partial T_M}{\partial i}\right)^2 \cdot \sigma_i^2 + \left(\frac{\partial T_M}{\partial a_2}\right)^2 \sigma_{a_2}^2 \qquad \text{when } \sigma_{K_2} = 0$$

$$= \bar{K}_2 \cdot \sigma_i^2 + 1 \cdot \sigma_{a_2}^2 = (6{,}25)^2 \cdot 9{,}32 \cdot 10^{-2} + (0{,}30)^2 \approx \underline{3{,}94\ °C}$$

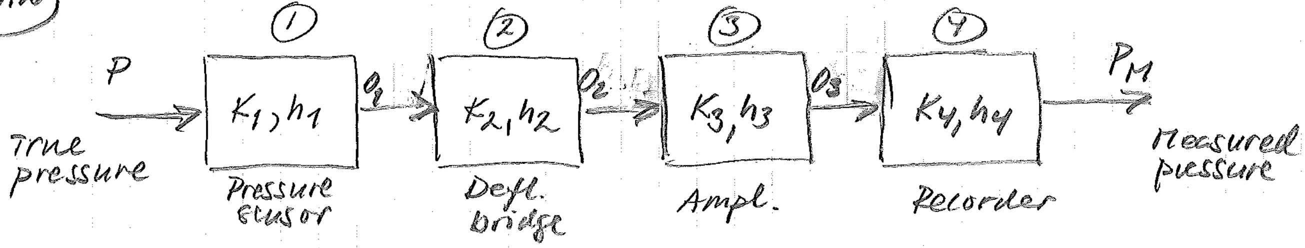
T_M is normal distributed $N(\bar{T}_M, \sigma_{T_M})$

$$\overline{\text{Error}} = \bar{T}_M - T = 116{,}57 - 117 = \underline{\underline{-0{,}43\ °C}}$$

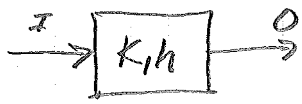
$$\sigma_{\text{Error}}^2 = \sigma_{T_M}^2 = 3.94\ (°C)^2 \Rightarrow \sigma_{\text{Error}} \approx \underline{\underline{1.99\ °C}}$$

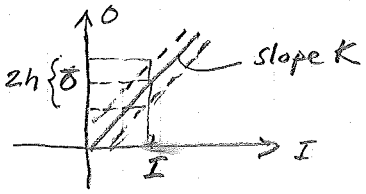
$$\left(\text{Error} = T_M - T \Rightarrow \underbrace{Var[\text{Error}]}_{\sigma^2_{\text{Error}}} = \underbrace{Var[T_M]}_{\sigma^2_{T_M}} + \underbrace{Var[T]}_{\sigma^2_T = 0} \right)$$

(3.2)



Error band model for each element!

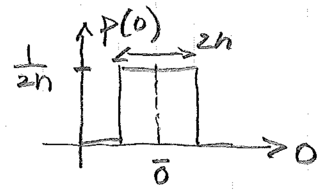




$O = \bar{O} \pm h$

$$\sigma_O^2 = \left(\frac{\partial O}{\partial I}\right)^2 \cdot \sigma_I^2 + \sigma_O^2$$

$$= K^2 \cdot \sigma_I^2 + \frac{(h)^2}{3}$$



Rect. distr.

$$\sigma_O^2 = \frac{(h)^2}{3}$$

a) Calculate σ_E ; $\sigma_E = \sigma_{P_M}$

(1) $K_1 = 10^{-4}\ \Omega/Pa$; $h_1 = \pm 0{,}005\ \Omega$

$$\sigma_{O_1}^2 = \underbrace{\left(\frac{\partial O_1}{\partial P}\right)^2 \sigma_P^2}_{=0} + \frac{(h_1)^2}{3} = \frac{(h_1)^2}{3} = \frac{(5 \cdot 10^{-3})^2}{3} = \frac{25 \cdot 10^{-6}}{3}$$

(2) $K_2 = 4 \cdot 10^{-2}\ mV/\Omega$; $h_2 = \pm 5 \cdot 10^{-4}\ mV$

$$\sigma_{O_2}^2 = \left(\frac{\partial O_2}{\partial O_1}\right)^2 \sigma_{O_1}^2 + \frac{(h_2)^2}{3} = K_2^2 \sigma_{O_1}^2 + \frac{(h_2)^2}{3} \approx 9{.}66 \cdot 10^{-8}$$

(3) $K_3 = 10^3$; $h_3 = \pm 0{,}5\ mV$

$$\sigma_{O_3}^2 = \left(\frac{\partial O_3}{\partial O_2}\right)^2 \sigma_{O_2}^2 + \frac{(h_3)^2}{3} = K_3^2 \sigma_{O_2}^2 + \frac{(h_3)^2}{3} \approx 1{,}8 \cdot 10^{-1}$$

(4) $K_4 = 250\ Pa/mV$; $h_4 = \pm 100\ Pa$

$$\sigma_{P_M}^2 = \left(\frac{\partial P_M}{\partial O_3}\right)^2 \sigma_{O_3}^2 + \frac{(h_4)^2}{3} = K_4^2 \sigma_{O_3}^2 + \frac{(h_4)^2}{3} = 1{,}46 \cdot 10^4\ [Pa^2]$$

$$\sigma_E = \sigma_{P_M} = \sqrt{1{.}46} \cdot 10^2 \approx \underline{120{,}8\ Pa}$$

3.2 cont.

b) $P = 5 \cdot 10^3$ Pa

$\bar{P}_M = (K_1 \cdot K_2 \cdot K_3 \cdot K_4) \cdot P$

$$\begin{cases} K_1 = 10^{-4}\ \Omega/\text{Pa} \\ K_2 = 4 \cdot 10^{-2}\ \text{mV}/\Omega \\ K_3 = 10^3 \\ K_4 = 225\ \text{Pa/mV} \quad \leftarrow \text{incorrectly adjusted} \end{cases}$$

$\bar{E} = \bar{P}_M - P = (K_1 \cdot K_2 \cdot K_3 \cdot K_4 - 1) \cdot P$

$= (10^{-4} \cdot 4 \cdot 10^{-2} \cdot 10^3 \cdot 225 - 1) \cdot P = -0.1 \cdot P$

$= -0.1 \cdot 5 \cdot 10^3 = \underline{\underline{-500\ \text{Pa}}}$

(3.7)

$$\underset{\text{True temperature}}{\Theta = 320\,K} \longrightarrow \underset{1)}{\boxed{\text{Sensor}}} \xrightarrow{R_\Theta} \underset{2)}{\boxed{\text{Bridge}}} \xrightarrow{V_0} \underset{3)}{\boxed{\text{Recorder}}} \longrightarrow \underset{\text{measured temp.}}{\Theta_M}$$

---

1) Thermistor (sensor).

Model eq: $\boxed{R_\Theta = K_1 \exp\left(\frac{\beta}{\Theta}\right)}$

$$\begin{cases} \bar{K}_1 = 5 \cdot 10^{-4}\ k\Omega & ; \ \sigma_{K_1} = 0{,}5 \cdot 10^{-4} \\ \bar{\beta} = 3 \cdot 10^{3}\ K & ; \ \sigma_\beta = 0 \end{cases}$$

---

2) Bridge

Model eq: $\boxed{V_0 = V_s \left(\frac{1}{1 + \frac{3{,}3}{R_\Theta}} - a_1 \right)}$

$\bar{V}_s = -3{,}00\ V$; $\sigma_{V_s} = 0{,}03$

$\bar{a}_1 = 0{,}77$; $\sigma_{a_1} = 0{,}01$

---

3) Recorder

Model eq: $\boxed{\Theta_M = K_2 \cdot V_0 + a_2}$

$\bar{K}_2 = 50{,}0\ K/V$; $\sigma_{K_2} = 0{,}0$

$\bar{a}_2 = 300\,K$; $\sigma_{a_2} = 3{,}0$

Mean Error : $\overline{Error} = \bar{\Theta}_M - \Theta$

1) $\bar{R}_{320} = \bar{K}_1 \exp\left(\frac{\bar{\beta}}{320}\right) = 5 \cdot 10^{-4} \exp\left(\frac{3 \cdot 10^3}{320}\right) = 5{,}895\ k\Omega$

2) $\bar{V}_0 = \bar{V}_s \left(\frac{1}{1 + \frac{3{,}3}{\bar{R}_{320}}} - \bar{a}_1 \right) = -3{,}0 \left(\frac{1}{1 + \frac{3{,}3}{5{,}895}} - 0{,}77 \right) \approx 0{,}387\ V$

3) $\bar{\Theta}_M = \bar{K}_2 \cdot \bar{V}_0 + \bar{a}_2 = 50{,}0 \cdot 0{,}387 + 300 \approx 319{,}35\ K$

and $\overline{Error} = 319{,}35 - 320 = \underline{\underline{-0{,}65\ K}}$

(3.7)

Cont. Standard deviation σ_E ; $\sigma_E = \sigma_{\Theta_M}$

1) $R_\theta = f(\theta, K_1, \beta)$

$$\sigma_{R_\theta}^2 = \left(\frac{\partial R_\theta}{\partial \theta}\right)^2 \underbrace{\sigma_\theta^2}_{=0} + \left(\frac{\partial R_\theta}{\partial K_1}\right)^2 \sigma_{K_1}^2 + \left(\frac{\partial R_\theta}{\partial \beta}\right)^2 \underbrace{\sigma_\beta^2}_{=0} = \left(\frac{\partial R_\theta}{\partial K_1}\right)^2 \sigma_{K_1}^2$$

$$= \left(\exp\left(\frac{\beta}{\theta}\right)\right)^2 \sigma_{K_1}^2 \cdot \left(\exp\left(\frac{3\cdot 10^3}{320}\right)\right)^2 \cdot (0.5\cdot 10^{-4})^2$$

$$\approx 3{,}4751 \cdot 10^{-1}$$

2) $V_0 = f(R_\theta, V_S, a_1)$

$$\sigma_{V_0}^2 = \left(\frac{\partial V_0}{\partial R_\theta}\right)^2 \sigma_{R_\theta}^2 + \left(\frac{\partial V_0}{\partial V_S}\right)^2 \sigma_{V_S}^2 + \left(\frac{\partial V_0}{\partial a_1}\right)^2 \sigma_{a_1}^2$$

× $$\frac{\partial V_0}{\partial R_\theta} = \frac{\partial}{\partial R_\theta}\left\{\frac{V_S \cdot R_\theta}{R_\theta + 3{,}3} - a_1 \cdot V_S\right\} = \frac{\partial}{\partial R_\theta}\left\{\frac{V_S \cdot R_\theta}{R_\theta + 3{,}3}\right\}$$

$$= \frac{(R_\theta + 3{,}3)\cdot V_S - 1(V_S \cdot R_\theta)}{(R_\theta + 3{,}3)^2} = \frac{(5{,}895 + 3{,}3)(-3) - ((-3)\cdot 5{,}895)}{(5{,}895 + 3{,}3)^2}$$

$$\approx 1{,}1709 \cdot 10^{-1}$$

× $$\frac{\partial V_0}{\partial V_S} = \left(\frac{1}{1 + \frac{3{,}3}{R_\theta}} - a_1\right) \approx \left(\frac{1}{1 + \frac{3{,}3}{5{,}895}} - 0.77\right) \approx 1{,}2889 \cdot 10^{-1}$$

× $$\frac{\partial V_0}{\partial a_1} = -V_S = 3.00$$

so $$\sigma_{V_0}^2 = (1.1709\cdot 10^{-1})^2 \cdot 3{,}4751\cdot 10^{-1} + (1{,}2889\cdot 10^{-1})^2 \cdot (0{,}03)^2 + (3.00)^2 \cdot (0{,}01)^2 \approx 5{,}6792 \cdot 10^{-3}$$

3) $\Theta_M = f(V_0, K_2, a_2)$

$$\sigma_{\Theta_M}^2 = \left(\frac{\partial \Theta_M}{\partial V_0}\right)^2 \sigma_{V_0}^2 + \left(\frac{\partial \Theta_M}{\partial K_2}\right)^2 \underbrace{\sigma_{K_2}^2}_{=0} + \left(\frac{\partial \Theta_M}{\partial a_2}\right)^2 \sigma_{a_2}^2$$

$$= K_2^2 \cdot \sigma_{V_0}^2 + 1 \cdot \sigma_{a_2}^2 = (50{,}0)^2 \cdot (5{,}6792\cdot 10^{-3})^2 + (3{,}0)^2 \approx 23{,}198$$

and $\sigma_E = \sigma_{\Theta_M} = \sqrt{23{,}198} \approx 4{,}82\ K$