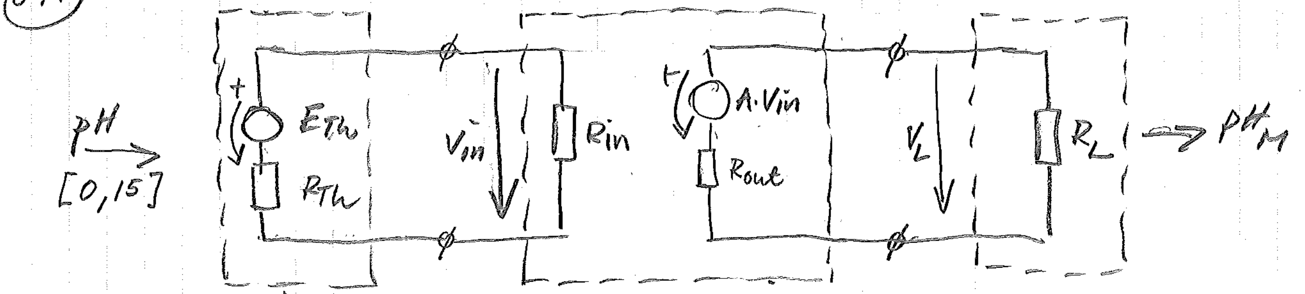
5.1



pH glass electrode
$\begin{cases} E_{Th} = 59\,\text{mV/pH} \\ R_{Th} = 10^9\,\Omega \end{cases}$

Amplifier
$\begin{cases} A = 1 \\ R_{out} = 100\,\Omega \\ R_{in} = ? \end{cases}$

Recorder
$R_L = 100\,\Omega$
$V_L \in [0, 100]\,\text{mV}$

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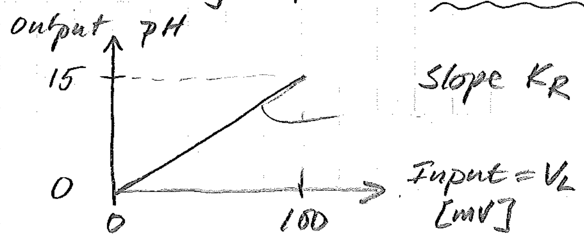
a) sensitivity of the recorder scale K_R



$$\text{Slope } K_R = \frac{\Delta O}{\Delta I} = \frac{15}{100} = 0{,}15\ \text{pH/mV}$$

$$pH_M = V_L \cdot K_R$$

$$= A \cdot V_{in} \left(\frac{R_L}{R_L + R_{out}}\right) \cdot K_R$$

$$= A \cdot \left(E_{Th} \cdot \frac{R_{in}}{R_{in} + R_{Th}}\right)\left(\frac{R_L}{R_L + R_{out}}\right) \cdot K_R$$

$$= A\,(59 \cdot pH)\left(\frac{R_{in}}{R_{in} + R_{Th}}\right)\left(\frac{R_L}{R_L + R_{out}}\right) \cdot K_R$$

$$= \underbrace{A \cdot 59 \left(\frac{R_{in}}{R_{in} + R_{Th}}\right)\left(\frac{R_L}{R_L + R_{out}}\right) \cdot K_R}_{=1 \text{ if no error}} \cdot pH$$

$$\Rightarrow K_R \cdot A \cdot 59 \left(\frac{R_{in}}{R_{in} + R_{Th}}\right)\underbrace{\left(\frac{R_L}{R_L + R_{out}}\right)}_{=\frac{1}{2}} = 1$$

$$\frac{R_{in}}{R_{in} + R_{Th}} = \frac{2}{K_R \cdot A \cdot 59} = \frac{2}{0{,}15 \cdot 59}$$

$$2(R_{in} + 10^9) = 0{,}15 \cdot 59 \cdot R_{in}$$

$$R_{in}(0{,}15 \cdot 59 - 2) = 2 \cdot 10^9 \Rightarrow R_{in} = 2{,}92 \cdot 10^8\ \Omega$$

5.1 cont.

b) $R_{Th}: 10^9 \Omega \curvearrowright 2 \cdot 10^9 \Omega$

$pH = 7$

$$pH_M = 59 \cdot pH \left(\frac{R_{in}}{R_{in} + R_{Th}} \right) \cdot \left(\frac{R_L}{R_L + R_{out}} \right) \cdot K_R$$

$$= 59 \cdot 7 \underbrace{\left(\frac{2{,}92 \cdot 10^8}{2{,}92 \cdot 10^8 + 2 \cdot 10^9} \right)}_{= 0{,}1274} \underbrace{\left(\frac{100}{100 + 100} \right)}_{= \frac{1}{2}} \cdot 0{,}15 \approx 3{,}95$$

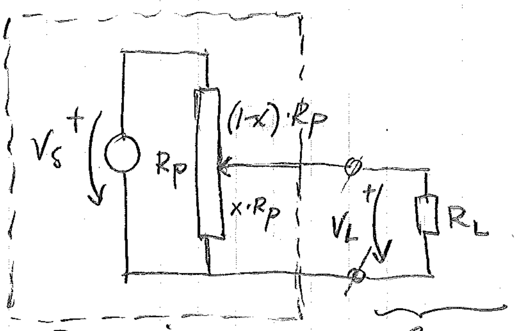
$\text{Error} = pH_M - pH = 3{,}95 - 7 = -3{,}05$

Percentage of full scale (= 15)

$$\frac{-3{,}05}{15} \cdot 100 = \underline{\underline{-20{,}3\%}}$$

5.2

The theory is presented in chapter 5.1.2 page 80 – 82.



Potentiometer displacement sensor

- $\ell = 25$ cm
- $P_{max} = 5W$
- $R_p \in [250, 2500]\,\Omega$ in steps of $250\,\Omega$

Recorder

- $R_L = 5000\,\Omega$

No loading: $V_{NL} = V_S \dfrac{x \cdot R_p}{R_p} = V_S \cdot x$ linear in x

Loading: $V_L = V_S \cdot x \cdot \dfrac{R_L}{R_p\,x(1-x) + R_L}$ non-linear in x

page 81-82

$\Rightarrow N(x) = V_{NL} - V_L$ max non-linearity

if $R_p / R_L \ll 1$ $\quad \hat{N} \approx 15 \cdot \dfrac{R_p}{R_L}$ % of full span ($= V_S$)

$\hat{N} < 2\%$ $\Rightarrow$ $15 \cdot \dfrac{R_p}{R_L} < 2$

$$R_p < \frac{2 \cdot R_L}{15} = \frac{2 \cdot 5000}{15} \approx 670\,\Omega$$

So $R_p < 670\,\Omega$ due to non-linearity error

Select R_p!

$R_p < 670\,\Omega$ and R_p in steps of $250\,\Omega$

$\Rightarrow R_p = 250 + 250 \cancel{+ 250}$

So R_p can be selected, according to the requirements $\hat{N} < 2\%$; to $250\,\Omega$ or $500\,\Omega$

5.2

cont.

- Maximum sensitivity is achieved when V_S is as large as possible
- But $P_{max} = 5W$ limit the possible value of V_S
- $$P = \frac{V_S^2}{R_P} \leq 5 \; (= P_{max}) \quad (*)$$

from (*): larger R_P gives higher V_S

$\Rightarrow$ select $R_P = 500\,\Omega$

So $$\frac{V_S^2}{500} \leq 5 \quad \Rightarrow \quad V_S \leq 50\ V$$

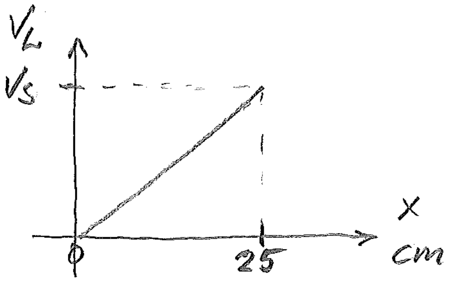
b) Design for maximum sensitivity

$\Rightarrow$ $R_P = 500\,\Omega$

$V_S = 50\,V$

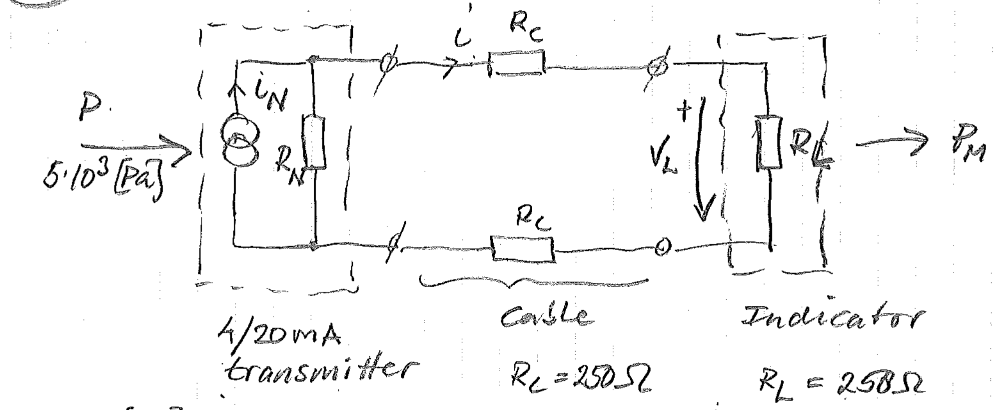
a) Maximum sensitivity (unloaded sensor V_L is linear in x)

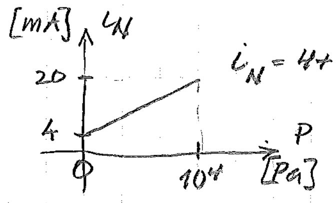
$V_L = K \cdot X$



$$K = \frac{V_S}{25} = \frac{50}{25} = 2\ V/cm$$

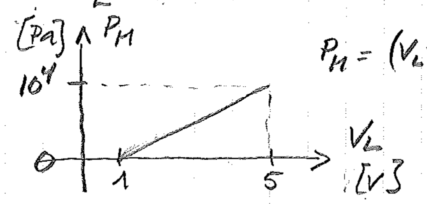
5.3





$i_N = 4 + \frac{16}{10^4} \cdot P$ [mA]

$R_N = 10^5\,\Omega$



$P_M = (V_L - 1) \cdot \frac{10^4}{4}$ [Pa]

When input pressure $P = 5 \cdot 10^3$ [Pa] compute P_M.

$$i_N = 4 + \frac{16}{10^4} \cdot 5 \cdot 10^3 = 4 + 8 = 12 \text{ mA}$$

$$V_L = i \cdot R_L = \left(i_N \cdot \frac{R_N}{R_N + 2R_C + R_L} \right) \cdot R_L$$

$$= 12 \cdot 10^{-3} \underbrace{\left(\frac{10^5}{10^5 + 750} \right)}_{\approx 0{,}9926} \cdot 250 \approx 2{,}978 \text{ V}$$

$$P_M = (V_L - 1)\frac{10^4}{4} = (2.978 - 1)\frac{10^4}{4} = 4{,}945 \cdot 10^3 \text{ [Pa]}$$

$$\text{Error} = P_M - P = (4{,}945 - 5) \cdot 10^3 = -0{,}055 \cdot 10^3$$
$$= -55 \text{ [Pa]}$$

Due to loading $i \neq i_N$

Less loading effect if $R_N \gg (2R_C + R_L)$

No loading $\Rightarrow i = i_N$