6.1

a) $\bar{y} = \frac{1}{N}\sum_{i=1}^{N} y_i \quad ; \quad N=50$

$$\Rightarrow \bar{y} = \frac{1}{50}\sum_{i=1}^{50} y_i$$

see MATLAB script P6-1.m

$$= \frac{1}{50}\left(-0{,}59 + 1{,}02 - 0{,}25 + \dots + 1{,}90\right) = \underline{\underline{0{,}15}}$$

$$\sigma = \sqrt{\frac{1}{N-1}\sum_{i=1}^{N}(y_i-\bar{y})^2} \quad ; \quad N=50$$

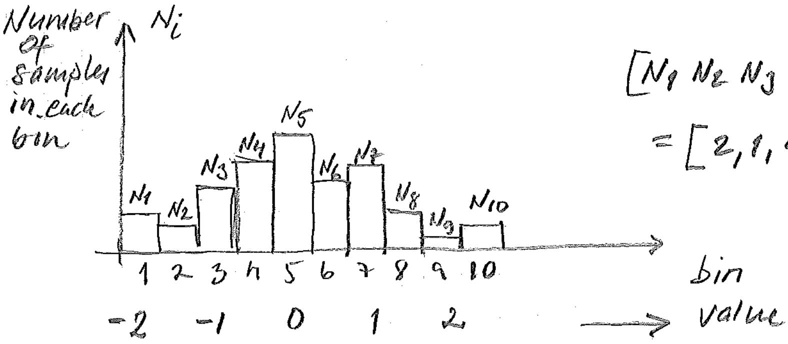
$$\Rightarrow \sigma = \sqrt{\frac{1}{49}\sum_{i=1}^{50}(y_i-\bar{y})^2}$$

$$= \sqrt{\frac{1}{49}\left[(-0{,}59-0{,}15)^2 + (1{,}02-0{,}15)^2 + \dots + (1{,}90-0{,}15)^2\right.}$$

$$\approx \underline{\underline{1{,}03}}$$

b) Calculate histogram ; $\Delta y = 0{,}5$

see MATLAB script P6-1.m Figure 1



$[N_1\ N_2\ N_3 \dots N_{10}]$

$= [2, 1, 4, 9, 13, 6, 8, 3, 1, 3]$

Number of bins is estimated as: $\frac{\max(y) - \min(y)}{\Delta y}$

$$= \frac{2{,}61 - (-2{,}26)}{0{,}5} \approx 9{,}74$$

$\Rightarrow$ say 10 bins

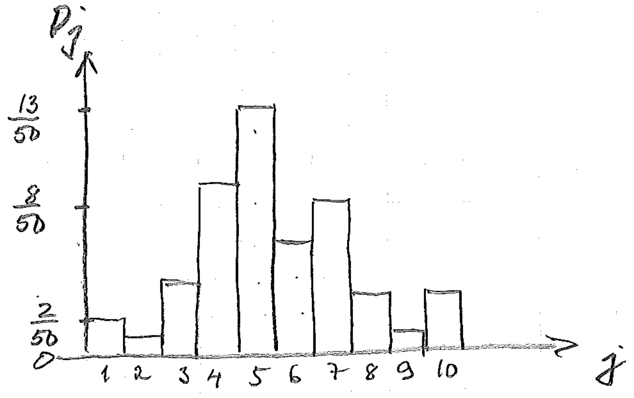
From the histogram the interval probability is computed as:

$$P_j = \frac{N_j}{N} \qquad j = 1, 2, \dots, 10$$

where $N = \sum_{i=1}^{10} N_i$ (total number of samples).

6.1 cont

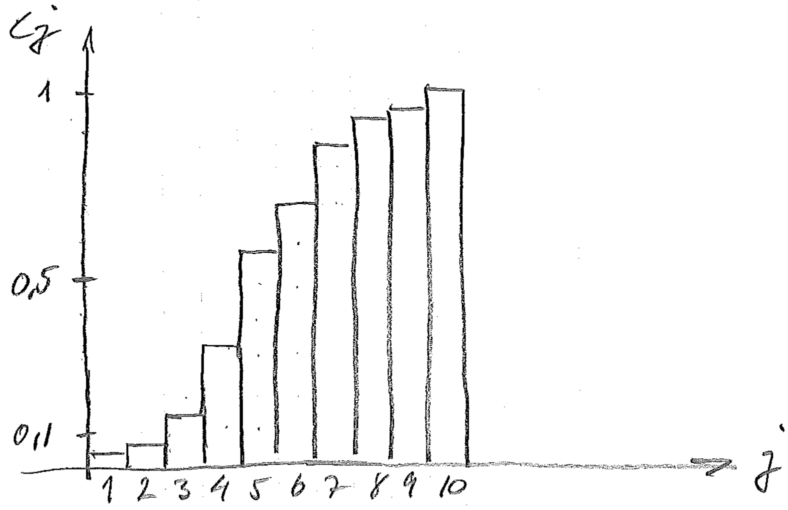
$$[P_1\ P_2\ \dots\ P_{10}] = \left[\frac{2}{50}, \frac{1}{50}, \frac{4}{50}, \frac{9}{50}, \frac{13}{50}, \frac{6}{50}, \frac{8}{50}, \frac{3}{50}, \frac{1}{50}, \frac{3}{50}\right]$$



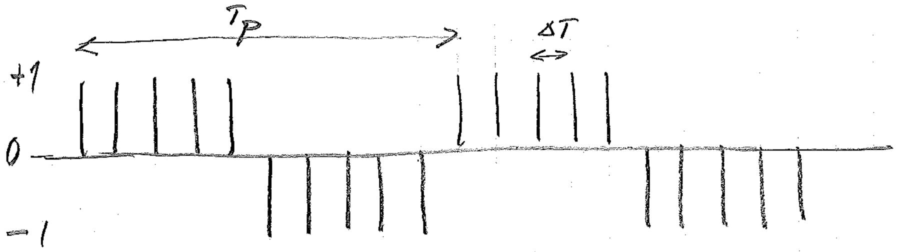
$$C_j = \sum_{j=1}^{k} P_k \qquad \text{Cumulative probability function}$$

$$\begin{cases} C_1 = P_1 \\ C_2 = P_1 + P_2 \\ C_3 = P_1 + P_2 + P_3 \\ \vdots \\ C_{10} = P_1 + P_2 + P_3 + P_4 + P_5 + P_6 + P_7 + P_8 + P_9 + P_{10} \end{cases}$$

$$\Rightarrow C_j = \left[\frac{2}{50}, \frac{3}{50}, \frac{7}{50}, \frac{16}{50}, \frac{29}{50}, \frac{35}{50}, \frac{43}{50}, \frac{46}{50}, \frac{47}{50}, \frac{50}{50}\right]$$



(6.2)



square wave signal y

$$R_{yy}(m\cdot\Delta T) = \frac{1}{10}\sum_{n=1}^{10} x(n)\,x(n-m\cdot\Delta T) \quad ; \; m = 0,1,2,\ldots,9,10$$

$$R_{yy}(0) = \frac{1}{10}\left(1\cdot1 + 1\cdot1 + 1\cdot1 + 1\cdot1 + 1\cdot1 + (-1)(-1) + (-1)(-1) + (-1)(-1) + (-1)(-1) + (-1)(-1)\right) = +1$$

$$R_{yy}(1) = \frac{1}{10}\left((-1)1 + 1\cdot1 + 1\cdot1 + 1\cdot1 + 1\cdot1 + (-1)1 + (-1)(-1) + (-1)(-1) + (-1)(-1) + (-1)(-1)\right) = +0.6$$

$$R_{yy}(2) = \frac{1}{10}\left((-1)\cdot1 + (-1)\cdot1 + 1\cdot1 + 1\cdot1 + 1\cdot1 + (-1)\cdot1 + (-1)\cdot1 + (-1)(-1) + (-1)(-1) + (-1)(-1)\right) = +0{,}2$$

$R_{yy}(3) = -0{,}2$

$R_{yy}(4) = -0{,}6$

$R_{yy}(5) = -1$

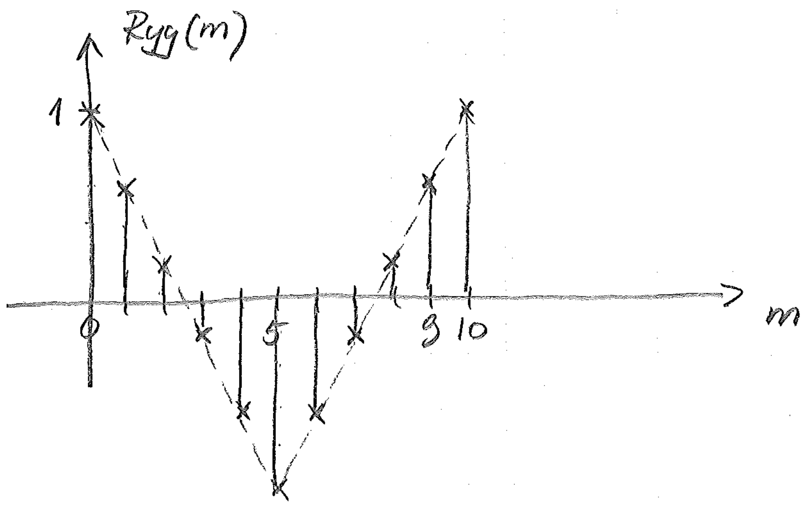
$R_{yy}(6) = -0{,}6$

$R_{yy}(7) = -0{,}2$

$R_{yy}(8) = +0{,}2$

$R_{yy}(9) = +0{,}6$

$R_{yy}(10) = +1$



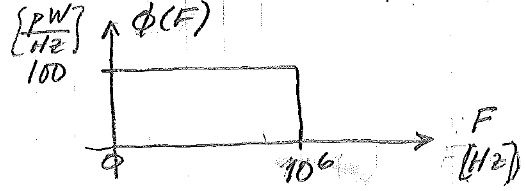
$R_{yy}(m)$ is also periodic with period $= T_p$ $(=10)$
(same period as the periodic signal y).

6.3

Signal $s(t) = A \cdot \sin(2\pi F \cdot t)$; $A = 1.4\,mV$, $F = 5\,kHz$

Noise $n(t)$: • Amplitude is Gaussian distributed with zero mean

• Uniform power spectrum



a) Noise signal $n(t)$; zero-mean process

- Total power $P_n = \int \Phi(F)\, dF = 100 \cdot 10^{-12} \cdot 10^{6} = 10^{-4}\ W$

- r.m.s value $n_{rms}^2 = P_n \Rightarrow n_{rms} = \sqrt{P_n} = 10^{-2}\ V$

- standard deviation: $\sigma_n^2 = n_{rms}^2$ if zero-mean process

$$\Rightarrow \sigma_n = n_{rms} = 10^{-2}\ V$$

b) $$SNR = 20\ {}^{10}\log\left(\frac{S_{rms}}{n_{rms}}\right)\ dB$$

where $\begin{cases} n_{rms} = 10^{-2}\ V & \text{(see a))} \\ S_{rms} = \frac{A}{\sqrt{2}} & \text{(sinusoidal signal)} \\ \quad = \frac{1.4}{\sqrt{2}} \cdot 10^{-3}\ V & \end{cases}$

$$\Rightarrow SNR = 20 \cdot {}^{10}\log\Bigg(\underbrace{\frac{\frac{1.4}{\sqrt{2}} \cdot 10^{-3}}{10^{-2}}}_{\approx 10^{-1}}\Bigg) \approx -20\ dB$$

6.3 cont.

c) $$y(t) = s(t) + n(t)$$

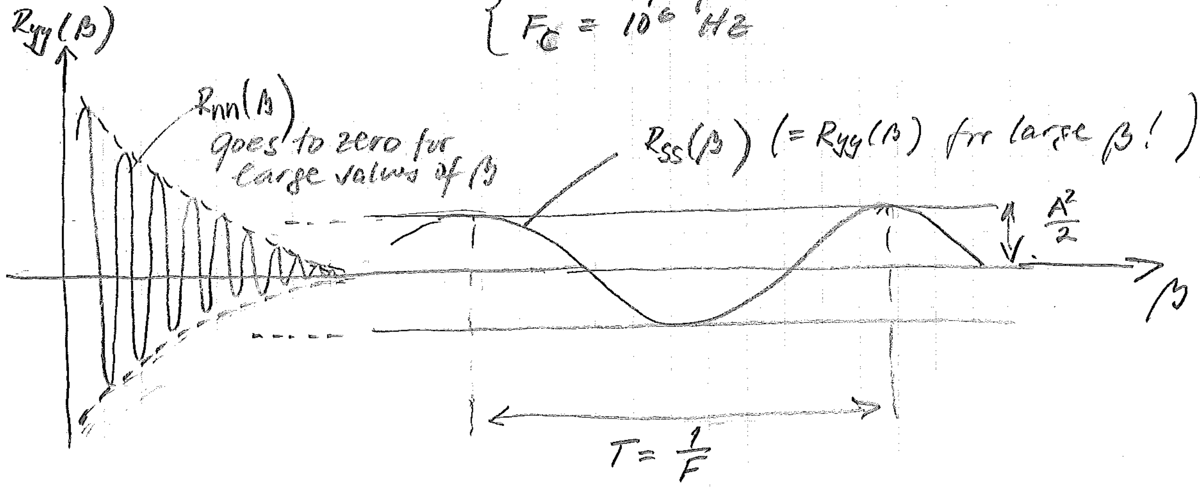
$$R_{yy}(\beta) = R_{ss}(\beta) + R_{nn}(\beta)$$ if $s(t)$ and $n(t)$ are uncorrelated

$$\begin{cases} R_{ss}(\beta) = \frac{A^2}{2}\cos(2\pi F\cdot\beta) & \begin{cases} A = 1.4\cdot10^{-3}\ \text{V} \\ F = 5\ \text{kHz} \end{cases} \\ R_{nn}(\beta) = P\,\frac{\sin(2\pi F_c\cdot\beta)}{\beta} & \end{cases}$$

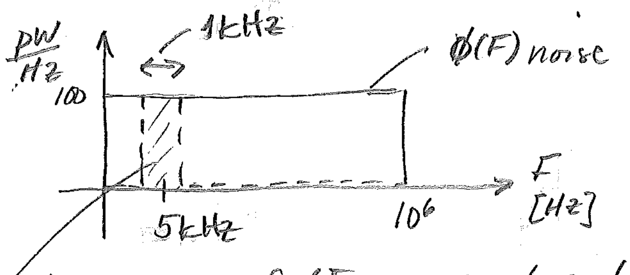
(see page 105; eq 6.27)

(see page 107; eq 6.31)

$$\begin{cases} P = 100\ \text{pW/Hz} \\ F_c = 10^6\ \text{Hz} \end{cases}$$



d)



bandpass filter: centre frequency = 5kHz
bandwidth = 1kHz

After filtering: $P_n = 100\cdot10^{-12}\cdot10^{3} = 10^{-7}\ \text{W}$

and $n_{rms} = \sqrt{P_n} = \sqrt{10^{-7}} = \sqrt{10\cdot10^{-8}}$
$= \sqrt{10}\cdot10^{-4}\ \text{V}$

$$SNR = 20\,{}^{10}\log\left(\frac{S_{rms}}{n_{rms}}\right) =$$

$$= 20\,{}^{10}\log\left(\frac{\frac{1.4}{\sqrt{2}}\cdot10^{-3}}{\sqrt{10}\cdot10^{-4}}\right) \simeq 9{,}9\ \text{dB}$$

6.3 cont.

e)

Averaging 100 sections

$$\Rightarrow \sigma_{AV} = \frac{\sigma}{\sqrt{100}} = \frac{\sigma}{10}$$ (see page 121 eq 6.63)

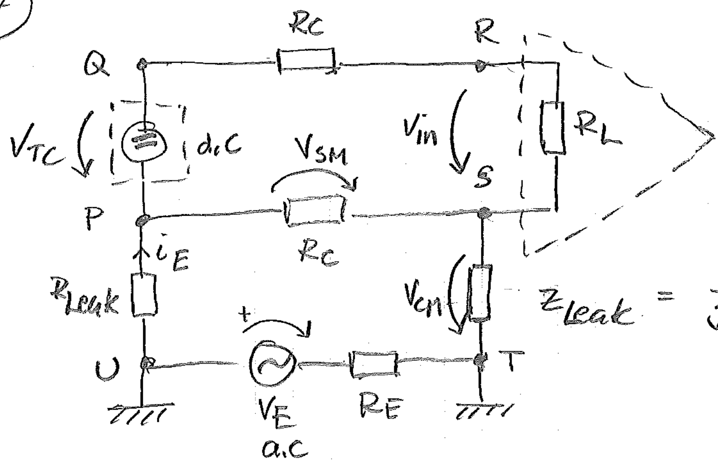
$n_{rms}^2 = \sigma^2$ if zero-mean process

$$\Rightarrow n_{rms}^{Filter+AV} = \frac{n_{rms}^{Filter}}{10} = \frac{\sqrt{10} \cdot 10^{-4}}{10} = \sqrt{10} \cdot 10^{-5}\ V$$

$$SNR = 20\ {}^{10}\log\left(\frac{s_{rms}}{n_{rms}^{Filter+AV}}\right)$$

$$= 20\ {}^{10}\log\left(\frac{\frac{1,4}{\sqrt{2}} \cdot 10^{-3}}{\sqrt{10} \cdot 10^{-5}}\right) \approx \underline{\underline{29,9\ dB}}$$

6.4

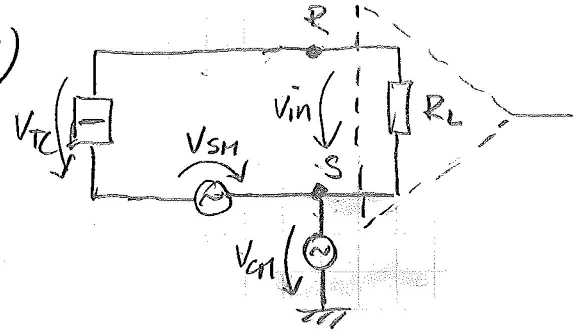


$$Z_{Leak} = \frac{1}{j\omega C_{Leak}}$$

Thermocouple voltage $V_{TC} = 10$ mV d.c
Cable resistance $R_C = 100\,\Omega$
Input resistance DVM $R_L = 10\,M\Omega$
Leakage resistance $R_{Leak} = 10\,\Omega$ (low imp. path)
Leakage capacitance $C_{Leak} = 1000\,pF = 10^{-9}\,F$
Earth potential difference $V_E = 100\,V_{rms}$; 50Hz
Earth resistance $R_E = 10\,\Omega$

Current flow in PQRS is negligible ($R_L \gg R_C$)
so i_E flows "only" in circuit UPST.

a)



$$\begin{cases} V_R = V_{CM} + V_{SM} + V_{TC} \\ V_S = V_{CM} \end{cases}$$

$$V_{in} = V_R - V_S = V_{TC} + V_{SM}$$

$$\begin{cases} V_{CM} = i_E\, Z_{Leak} \\ i_E = \dfrac{V_E}{(R_{Leak} + R_C + Z_{Leak} + R_E)} \\ Z_{Leak} = \dfrac{1}{j\omega C_{Leak}} = -j\dfrac{1}{2\pi F \cdot C_{Leak}} \simeq -j\,3{,}18\cdot 10^6\,\Omega \end{cases}$$

$$\Rightarrow V_{CM} = \left(\frac{100}{10 + 100 - j3{,}18\cdot 10^6 + 10}\right)(-j\,3{,}18\cdot 10^6)$$

$$= \left(\frac{100}{120 - j3{,}18\cdot 10^6}\right)(-j\,3{,}18\cdot 10^6) \Rightarrow V_{CM}^{rms} \approx 100\,V$$

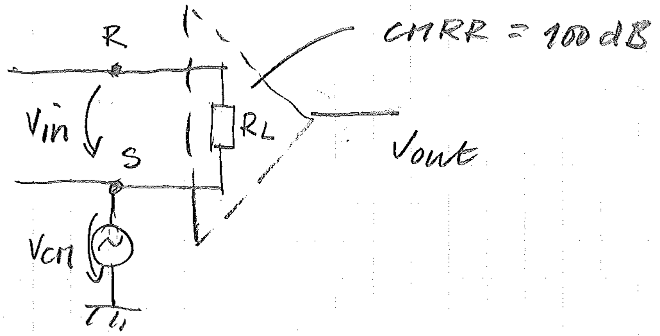
6.4 cont.

$$V_{SM} = i_E \cdot R_C$$

$$= \frac{100}{(10+100-j\,3{,}18\cdot 10^6+10)} \cdot 100 = \frac{10^4}{(120-j\,3{,}18\cdot 10^6)}$$

$$\Rightarrow V_{SM}^{rms} \approx \frac{10^4}{3.18\cdot 10^6} \approx 3{,}15 \text{ mA}$$

b)



$$V_{out} = A_{DM} \cdot V_{in} + A_{CM} \cdot V_{CM}$$

where $\begin{cases} A_{DM} \text{ is assumed to be } = 1 \\ CMRR = 20\,{}^{10}\log\left(\frac{A_{DM}}{A_{CM}}\right) = 100 \text{ dB} \quad (*) \end{cases}$

$$(*) \Rightarrow \quad {}^{10}\log\left(\frac{A_{DM}}{A_{CM}}\right) = \frac{100}{20} = 5$$

$$\Rightarrow \frac{A_{DM}}{A_{CM}} = 10^5$$

and if $A_{DM} = 1 \Rightarrow A_{CM} = 10^{-5}$ ggr.

$$V_{DM} = V_{in} = V_{TC} \pm \hat{V}_{SM} = V_{TC} \pm V_{SM}^{rms} \cdot \sqrt{2} = 10 \pm 3{,}1\cdot\sqrt{2} \text{ mV}$$

$$= 10 \pm 4{,}4 \text{ mV}$$

$$V_{CM} = \pm \hat{V}_{CM} = \pm V_{CM}^{rms} \cdot \sqrt{2}$$

$$= \pm 100 \cdot \sqrt{2} = \pm 141 \text{ V}$$

$$V_{out_{MAX}} = V_{in_{MAX}} + A_{CM} \cdot V_{CM_{MAX}} = 14{,}4\cdot 10^{-3} + 141 \cdot 10^{-5}$$

$$= 14{,}4 + 1{,}41 = 15{,}8 \text{ mV}$$

$$V_{out_{MIN}} = V_{in_{MIN}} + A_{CM} \cdot V_{CM_{MIN}} = 5{,}6 \cdot 10^{-3} - 141 \cdot 10^{-5}$$

$$= 5{,}6 - 1{,}41 \approx \underline{\underline{4{,}2 \text{ mV}}}$$

6.5

$y(t) = s(t) + n(t)$

where:

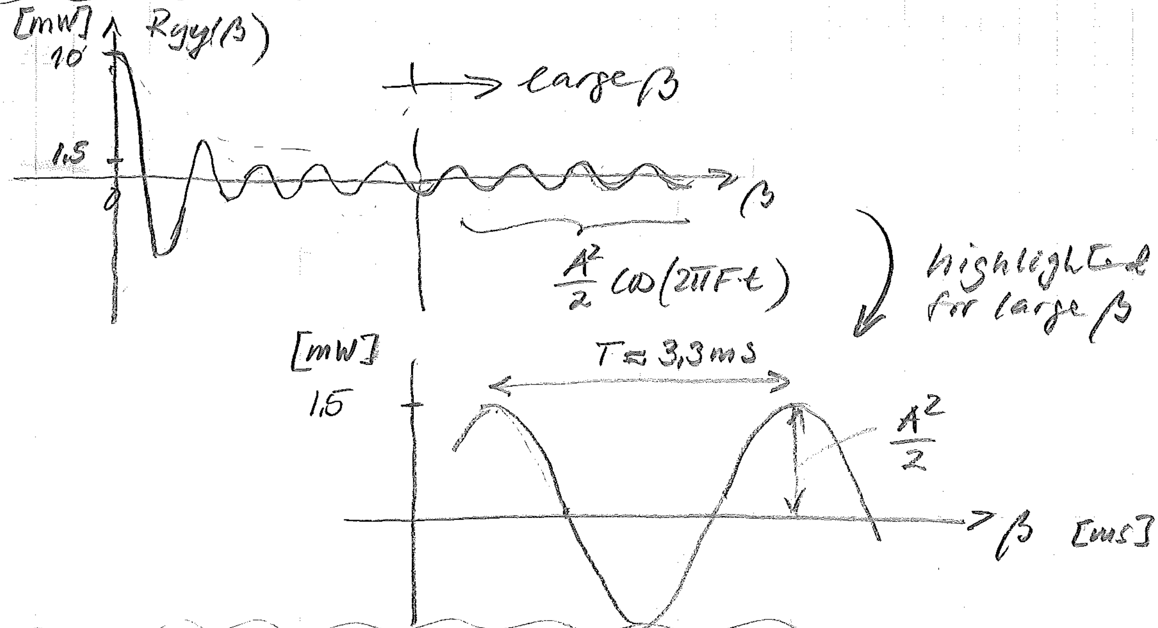
$s(t) = A \cdot \sin(2\pi F \cdot t)$

$n(t) =$ is random noise

$R_{yy}(\beta) = R_{ss}(\beta) + \underbrace{R_{nn}(\beta)}_{\text{go to zero for large } \beta}$

$R_{ss}(\beta) = \frac{A^2}{2} \cos(2\pi F \cdot t)$

So for large β $\quad R_{yy}(\beta) = R_{ss}(\beta)$



When β is large; i.e $R_{yy}(\beta) = R_{ss}(\beta)$

d) signal amplitude A

$\frac{A^2}{2} = 1,5$ mW from highlighted figure $R_{yy}(\beta)$

$\Rightarrow A^2 = 3,0$ mW and $A \approx 55$ mV

e) Signal frequency F

$T = 3,3$ ms from highlighted figure $R_{yy}(\beta)$

$\Rightarrow F = \frac{1}{T} = \frac{1}{3,3 \cdot 10^{-3}} \approx 300$ Hz

6.5 cont.

a) Signal power P_s

$P_s = R_{ss}(0)$

$R_{ss}(0)$ is the highest value of $R_{yy}(\beta)$

$\Rightarrow P_s = 1{,}5 \text{ mW}$

b) Noise power $P_n = R_{nn}(0)$

$R_{yy}(0) = R_{ss}(0) + R_{nn}(0) \Rightarrow R_{nn}(0) = R_{yy}(0) - R_{ss}(0)$

$= 10 - 1{,}5 = 8{,}5 \text{ mW}$

c) SNR in dB

$$SNR = 10 \cdot {}^{10}\log\left(\frac{P_s}{P_n}\right)$$

$$= 10\, {}^{10}\log\left(\frac{1{,}5}{8{,}5}\right) \simeq -7{,}5 \text{ dB}$$

f) Noise standard deviation (when zero-mean is assumed)

$\sigma_n^2 = P_n$ when zero-mean process

$$\Rightarrow \sigma_n = \sqrt{P_n} = \sqrt{8{,}5 \cdot 10^{-3}} = \sqrt{85 \cdot 10^{-4}}$$

$$\approx 92{,}2 \text{ mV}$$