Using stochastic geometry to model packet reception probabilities in vehicular networks

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Motivation

Safety

Efficiency
Motivation

To achieve optimal and safe coordination we need to understand and be able to handle uncertainties due to communication.
Outline

- V2X Communication
- Stochastic Geometry and Interference
- Example: Packet Reception Probability
- Recent Results
- Summary & Outlook
V2X communication

- Spectrum allocated in 5.9 GHz band

- Standards and technologies
  - IEEE 802.11p
    - PHY: Orthogonal frequency division multiplexing (OFDM) in 10 MHz channels
    - MAC: Carrier sense multiple access (CSMA)
    - No association with central access point
  - 5G device-to-device (D2D)

- Challenges
  - Guaranteed reliability
  - Constantly changing topologies
  - Harsh and varying propagation environment
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Stochastic Geometry and Interference

- Why stochastic geometry
  - Average system behavior in closed form

- Location of interfering transmitters are modeled using point processes

- The Poisson point process (PPP)
  - Characterized by single parameter \( \lambda \)
  - No. points in set \( B \in \mathbb{R}^n \) is Poisson distributed with mean \( \lambda |B| \)

- How does our work differ
  - Roads impose restrictions
  - 2D scenario but interferes along lines

- Limitations
  - Correlation
  - Fading distributions
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Packet reception probability

\[ P_r = P_t S l(d) + I + P_{\text{noise}} \]

\[ \text{SINR} = \frac{P_t S l(d)}{I + P_{\text{noise}}} \]
Packet reception probability

- Abstract the vehicles as points in a one dimensional space

- Consider the case when
  - Path loss function \( l(x_{tx}, x_{rx}) = A|x_{rx} - x_{tx}|^{-\alpha} \) with \( \alpha = 2 \)
  - Fading \( S \sim \exp(1) \)
  - Interferers according to homogeneous PPP \( \Phi \) with density \( \lambda \)
  - Packet is successfully decoded if SINR is above a certain threshold \( \beta \)
Packet reception probability

- Success probability

\[ P(\beta, x_{tx}, x_{rx}) = \Pr \left[ \frac{S}{I + P_{\text{noise}}} > \beta \right] \]

where \[ I = \sum_{x \in \Phi} P_t S_x l(x, x_{rx}) \]

- We need to average over fading on useful link and interference

\[ P(\beta, x_{tx}, x_{rx}) = \mathbb{E}_I \left\{ \Pr \left( S_0 > \left( I + P_{\text{noise}} \right) \frac{\beta}{P_t l(x_{tx}, x_{rx})} \right) \right\} \]

Two interpretations! Interference distribution Interfering vehicle at location \( x \)
Packet reception probability

- For exponential fading, the CCDF has the form $\bar{F}_{S_0}(s_0) = e^{-s_0}$

\[
P(\beta, x_{tx}, x_{rx}) = \int_0^\infty e^{-(t+P_{\text{noise}})\beta} f_I(t) \, dt
\]

To find the success probability we need to evaluate the Laplace transform!

Definition of Laplace transform:

\[
\int_0^\infty \int_0^\infty e^{-tf} f(t) \, dt \, dt
\]
Packet reception probability

- Evaluating the Laplace transform of the interference distribution

\[ \mathcal{L}_{f_I}(\tilde{\beta}) = \mathbb{E} \left[ \prod_{x \in \Phi} \exp \left( -\tilde{\beta} P_t S_x A |x - x_{rx}|^{-\alpha} \right) \right] \]

\[ = \mathbb{E}_\Phi \left[ \prod_{x \in \Phi} \mathbb{E}_{S_x} \left\{ \exp \left( -\tilde{\beta} P_t S_x A |x - x_{rx}|^{-\alpha} \right) \right\} \right] \]

\[ = \mathbb{E}_\Phi \left[ \prod_{x \in \Phi} \frac{1}{1 + \tilde{\beta} P_t A |x - x_{rx}|^{-\alpha}} \right] \]

\[ = \exp \left( -\lambda \int_{-\infty}^{\infty} \frac{1}{1 + |x - x_{rx}|^{\alpha}/\tilde{\beta} P_t A} \, dx \right) \]

\[ = \exp \left( \frac{-\lambda 2\pi}{\alpha} \left( \tilde{\beta} P_t A \right)^{\frac{1}{\alpha}} \csc \left( \frac{\pi}{\alpha} \right) \right) \]

We don’t know the interference distribution, but using tools from stochastic geometry we can evaluate its Laplace transform.

Probability generating functional (PGFL):

\[ \mathbb{E}_\Phi \left[ \prod_{x \in \Phi} \nu(x) \right] = \exp \left( -\int_{\mathbb{R}^d} (1 - \nu(x)) \lambda(\,dx) \right) \]
Packet reception probability

- Final expression for success probability

\[
P(\beta, x_{tx}, x_{rx}) = \exp\left(-\frac{P_{\text{noise}} B_{\beta} |x_{rx} - x_{tx}|^2}{P_t A}\right) \exp\left(-\frac{-\lambda 2\pi \beta^{1/\alpha} |x_{rx} - x_{tx}|}{\alpha} \csc\left(\frac{\pi}{\alpha}\right)\right)
\]

- Numerical results (outage prob.)

- Transmit power: \( P_t = 100 \text{ mW} \)
- Noise power: \( P_{\text{noise}} = -99 \text{ dBm} \)
- SINR threshold: \( \beta = 8 \text{ dB} \)
- Path loss exp: \( \alpha = 2 \)
- Path loss coeff: \( A = 0.0025 \)

\[
\mathcal{L}_{f_I}(\tilde{\beta}) = \exp\left(-\frac{-\lambda 2\pi \tilde{\beta}^{1/\alpha}}{\alpha} \left(\frac{\tilde{\beta} P_t A}{\alpha}\right) \csc\left(\frac{\pi}{\alpha}\right)\right)
\]

Recall:

\[
\text{distance between receiver and transmitter, } |x_{rx} - x_{tx}| [\text{m}]
\]

\[
\text{outage probability, } P_{\text{out}}(\beta, x_{tx}, x_{rx})
\]

\[
\text{outage probability, } P_{\text{out}}(\beta, x_{tx}, x_{rx})
\]
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Recent Results

- Simple expression for success probability in 4-lane intersection scenario given:
  - Small scale fading
  - Aloha MAC
  - PPPs with homogeneous density

\[
\mathbb{P}(\beta) = \Pr [\text{SINR} > \beta] = \exp \left( \frac{-\beta r_{\text{tx}}^\alpha P_{\text{noise}}}{AP_t} \right) \quad \text{interference free}
\]

\[
\times \exp \left( -p\lambda_1 \sqrt{\beta r_{\text{tx}}^\pi} \right) \quad \text{own road}
\]

\[
\times \exp \left( -p\lambda_2 \pi r_{\text{tx}}^2 \beta \right) \quad \text{parallel road}
\]

\[
\times \exp \left( -p\lambda_3 \pi r_{\text{tx}}^2 \beta \right) \quad \text{near perpendicular road}
\]

\[
\times \exp \left( -p\lambda_4 \pi r_{\text{tx}}^2 \beta \right) \quad \text{far perpendicular road}
\]
Recent Results

- **General procedure to determine**
  - Packet reception probability for a selected link
  - System wide throughput

- **Model repository to model different environments**
  - Small scale fading / large scale fading (shadowing)
  - Aloha/CSMA MAC
  - Loss function to model line of sight/ non line of sight
Recent Results

- Example results

<table>
<thead>
<tr>
<th>MAC</th>
<th>Fading</th>
<th>Loss Function</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aloha</td>
<td>Exponential</td>
<td>Euclidian</td>
</tr>
<tr>
<td>CSMA</td>
<td>Erlang</td>
<td>Manhattan</td>
</tr>
</tbody>
</table>

The figure shows the outage probability $P_{out}(\beta, x_{rx}, x_{tx})$ as a function of the distance between the receiver and transmitter $\|x_{rx} - x_{tx}\|_2$ for different MAC protocols and fading models. The graphs compare analytical and simulated results for exponential and Erlang fading distributions.
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Conclusions & Outlook

• Need to
  – Understand and be able to handle uncertainties due to imperfect communication

• We present
  – General framework with model repository

• Possible Future work
  – Simplify the models by using stochastic equivalence to incorporate channel properties into the geometry
  – Combined analysis of communication and control

E. Steinmetz, M. Wildemeersch and H. Wymeersch

[C] A Stochastic Geometry Model for Vehicular Communication near Intersections
E. Steinmetz, M. Wildemeersch, T.Q.S. Quek and H. Wymeersch

[D] Reception Probabilities in 5G Vehicular Communications close to Intersections
E. Steinmetz, M. Wildemeersch, T.Q.S. Quek and H. Wymeersch